PROBLEM:

The Euler and inverse Euler formulas can often simplify a messy complex formula.

(a) Evaluate: $(\cos(\pi/3) + j\sin(\pi/3))^3$. Give the answer in rectangular form.

(b) Simplify the following expression

$$z(t) = \frac{e^{j\omega t} - e^{-j\omega t}}{e^{j\omega t} + e^{-j\omega t}}$$

by giving a simple formula for the magnitude and phase of $z(t)$.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $\left[\underbrace{\cos(\pi/3) + j\sin(\pi/3)}\right]^3$

This is Euler's formula for $e^{j\pi/3}$

$$\left[e^{j\pi/3}\right]^3 = e^{j3\pi/3} = e^{j\pi} = -1 + j0$$

(b) $$z(t) = \frac{e^{j\omega t} - e^{-j\omega t}}{e^{j\omega t} + e^{j\omega t}}$$

Use inverse Euler:

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \quad \& \quad \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$z(t) = \frac{2j\sin(\omega t)}{2\cos(\omega t)} = j\,\frac{\sin(\omega t)}{\cos(\omega t)}$$

$$= e^{j\pi/2}\tan(\omega t)$$

phase is $\pi/2$

"Magnitude" is $\tan(\omega t)$

Strictly speaking, the magnitude is the absolute value of $\tan(\omega t)$ since tangent can be negative.

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