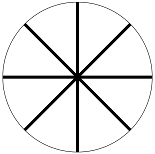
PROBLEM:



In old TV movies, all of us have seen the phenomenon where a spoked wagon wheel appears to move backwards. This is due to the 30 frames/sec sampling rate used in transmitting TV images. In the figure to the right, an eight-spoked wheel is shown. Assume that the wheel is rotating *clockwise at a constant speed* of 4 rev/sec.

(a) Write a rotating phasor formula for the observed movement of an individual spoke. This should be a continuous-time signal formula that depends on t (in sec).

(b) Given the sampling done by TV scanning, write a formula for the movement of an individual spoke as a function of the frame index n.

(c) Determine what a TV viewer will see, i.e., the observed rotation rate as a function of time t. Give your answer as the number of rotations per second and the direction of rotation.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $p(t)$ = position of spoke that is horizontal at $t=0$

clockwise = negative freq.

$$p(t) = re^{-j2\pi(4)t} = re^{-j8\pi t}$$

(b) Sampling at $F_s = 30$ frames/sec $\Rightarrow$ $t \leftrightarrow n/F_s$

$$p[n] = p(t)\Big|_{t=n/30} = re^{-j2\pi(4)n/30} = re^{-j2\pi(\frac{2}{15})n}$$

(c) Aliasing occurs at multiples of $2\pi/8$ because there are 8 spokes.

The digital freq is $\hat{\omega} = -2\pi(\frac{2}{15})$

Now we must add/subtract multiples of $2\pi/8$ to get the lowest possible freq.

$$-2\pi\left(\frac{2}{15}\right) + 2\pi\left(\frac{1}{8}\right)\ell$$

$$= 2\pi\left(-\frac{2}{15} + \frac{\ell}{8}\right)$$

Possible freqs are: $\hat{\omega} = 2\pi\left(\frac{-2}{15}\right),\ 2\pi\left(\frac{-1}{120}\right),\ 2\pi\left(\frac{14}{120}\right)$

SMALLEST FREQ.

The freq $\hat{\omega} = 2\pi\left(\frac{-1}{120}\right)$ converts back to analog as $F_A = \frac{F_s\hat{\omega}}{2\pi}$

$$F_A = \frac{30 \times 2\pi \times (-1/120)}{2\pi} = -1/4 \text{ rev/sec}$$

Appears to make 1 clockwise revolution every 4 seconds. IF YOU CAN FOLLOW ONE SPOKE