PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = -0.9\,y[n-1] + 3x[n] - 3x[n-1] \qquad \text{(DIFFERENCE EQUATION)}$$

(a) Find the z-transform operator representation $H(z)$ for the system in the difference equation.

(b) Find the poles and zeros of the system and plot their location in the z-plane.

(c) Determine the impulse response: give a formula.

(d) Plot the impulse response versus n.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $y[n] = -0.9\,y[n-1] + 3\,x[n] - 3\,x[n-1]$

$a_1 = -0.9 \qquad b_0 = 3 \qquad b_1 = -3$

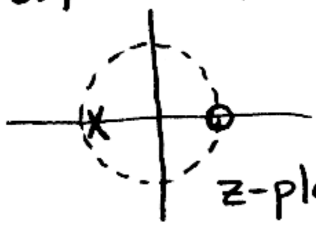
$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{3 - 3z^{-1}}{1 + 0.9z^{-1}}$$

(b) Multiply top & bottom by z

$$H(z) = \frac{3z-3}{z+0.9}$$

ZERO: $3z - 3 = 0 \Rightarrow z = 1$

POLE: $z + 0.9 = 0 \Rightarrow z = -0.9$



z-plane

(c) Impulse Response:
(Make a table)

n	<0	0	1	2	3	4	...
$x[n] = \delta[n]$	0	1	0	0	0	0	...
$y[n] = h[n]$	0	3	-5.7	5.13	-4.617	4.15	...

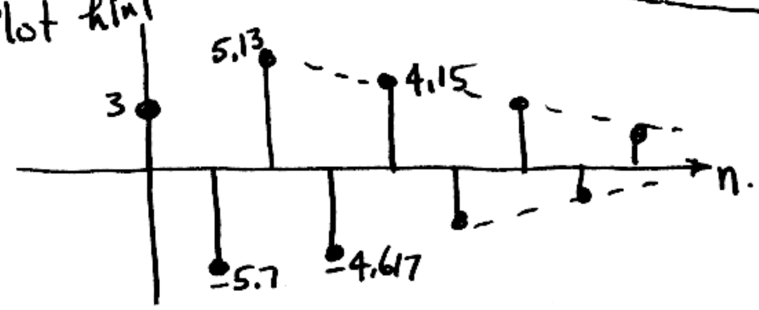
$y[2] = -0.9\,y[1] = -0.9(-5.7) = 5.13$

$y[1] = -0.9\,y[0] + 3\,x[1] - 3\,x[0] = -2.7 + 3(0) - 3 = -5.7$

for $n \geq 2$, $y[n] = -0.9\,y[n-1]$

FORMULA: $$y[n] = \begin{cases} 0 & \text{for } n < 0 \\ 3 & n = 0 \\ -5.7(-0.9)^{n-1} & \text{for } n \geq 1 \end{cases}$$

(d) Plot $h[n]$



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