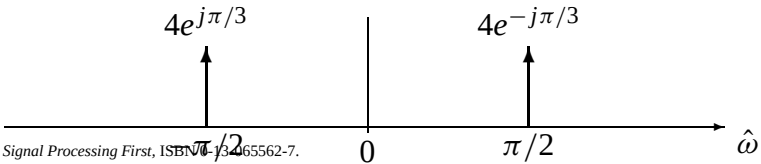
PROBLEM:

A feedback filter is characterized by the following frequency response:

$$\mathcal{H}(\hat{\omega}) = \frac{5}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$$

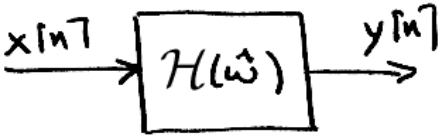
If the input to the filter is a sinusoidal signal $x[n]$ with the following spectrum, determine the output, $y[n]$ for $-\infty < n < \infty$. Give a formula for $y[n]$ versus n.



SPECTRUM OF $x[n]$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



If $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$

then $y[n] = A|H(\hat{\omega}_0)|\cos(\hat{\omega}_0 n + \varphi + \angle H(\hat{\omega}_0))$

So we need MAG & PHASE of $H(\hat{\omega})$ at $\hat{\omega}_0$

In this case, $x[n] = 8\cos(\frac{\pi}{2}n - \frac{\pi}{3}) \Rightarrow \hat{\omega}_0 = \pi/2$

$$H(\pi/2) = \frac{5}{1 - \frac{1}{2}e^{-j\pi/2}} = \frac{5}{1 + \frac{1}{2}j} = 4.47e^{-j0.148\pi}$$

$$\Rightarrow y[n] = (8)(4.47)\cos\left(\frac{\pi}{2}n - \frac{\pi}{3} - 0.148\pi\right)$$

$$= 35.78\cos\left(\frac{\pi}{2}n - 0.481\pi\right)$$