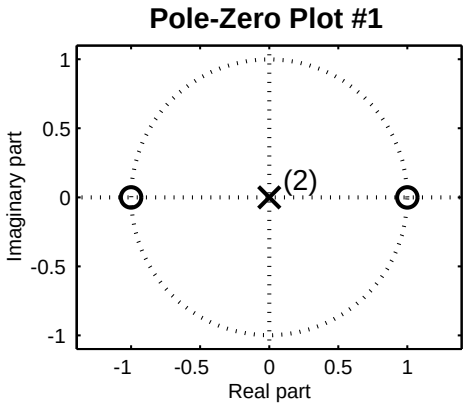
PROBLEM:

Questions about the frequency response of an FIR filter:

(a) Determine a formula for the frequency response of an FIR filter defined by the pole-zero plot below:



(b) For the FIR filter in part (a), write a simplified version of the frequency response $H(e^{j\hat{\omega}})$ and use it to prove that the maximum value of the frequency response magnitude will be at $\hat{\omega} = \pm\pi/2$. If convenient, draw a sketch of $|H(e^{j\hat{\omega}})|$.

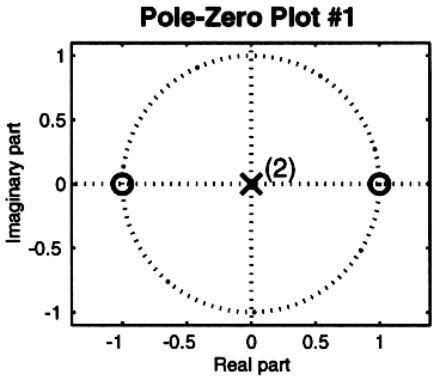
(c) The pole-zero plot does not define the scaling of the frequency response. Therefore, you can rescale $H(e^{j\hat{\omega}})$ with a scaling constant β so that the *maximum* value of the frequency response $\beta H(e^{j\hat{\omega}})$ will be equal to one. Determine the numerical value of the scaling constant β for the frequency response from parts (a) and (b).

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Questions about the frequency response of an FIR filter:

(a) Determine a formula for the frequency response of an FIR filter defined by the pole-zero plot below:



$$H(z) = \beta \frac{(z-1)(z+1)}{z^2}$$

$$= \beta(1-z^{-1})(1+z^{-1})$$

$$= \beta(1-z^{-2})$$

$$H(e^{j\hat{\omega}}) = \beta(1-e^{-j2\hat{\omega}})$$

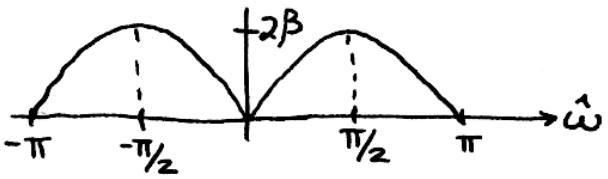
(b) For the FIR filter in part (a), write a simplified version of the frequency response $H(e^{j\hat{\omega}})$ and use it to prove that the maximum value of the frequency response magnitude will be at $\hat{\omega} = \pm\pi/2$. If convenient, draw a sketch of $|H(e^{j\hat{\omega}})|$.

$$H(e^{j\hat{\omega}}) = \beta e^{-j\hat{\omega}}\left(e^{+j\hat{\omega}} - e^{-j\hat{\omega}}\right) = \beta e^{-j\hat{\omega}} 2j \sin\hat{\omega}$$

$$|H(e^{j\hat{\omega}})| = 2\beta|\sin\hat{\omega}|$$

MAX of $\sin\hat{\omega}$ is at $\hat{\omega} = \pi/2$

MAX VALUE IS 2β



(c) The pole-zero plot does not define the scaling of the frequency response. Therefore, you can rescale $H(e^{j\hat{\omega}})$ with a scaling constant β so that the *maximum* value of the frequency response $\beta H(e^{j\hat{\omega}})$ will be equal to one. Determine the numerical value of the scaling constant β for the frequency response from parts (a) and (b).

In part (b) we found the max to be 2β.

Thus, choosing $\beta = 1/2$ will make the maximum equal to one.