PROBLEM:

Some miscellaneous complex number problems:

(a) Evaluate the complex number $z = \dfrac{j^{-1} - j^{-2}}{j^{-3} + j^{-4}}$.

(b) The following set defines a curve in the complex plane (i.e., the 2-D plane): $\{z : |z - j| = 2\}$. Draw a sketch of this curve.

(c) The following set defines a curve in the complex plane (i.e., the 2-D plane): $\{z : \Im m\{z\} = \pi\}$. Draw a sketch of this curve.

SOLUTION

a) $z = \dfrac{j^{-1} - j^{-2}}{j^{-3} + j^{-4}} = \dfrac{\frac{1}{j} - \frac{1}{j^2}}{\frac{1}{j^3} + \frac{1}{j^4}}$

NOTE:
$j = \sqrt{-1} = j$
$j^2 = \sqrt{-1}\sqrt{-1} = -1$
$j^3 = \sqrt{-1} \cdot -1 = -j$
$j^4 = (-1)^2 = 1$

$\quad = \dfrac{-j+1}{(+j+1)}\left(\dfrac{j-1}{j-1}\right) = \dfrac{2}{-2} = -1$

$\dfrac{1}{j} = \dfrac{1}{\sqrt{-1}}\left(\dfrac{\sqrt{-1}}{\sqrt{-1}}\right) = \dfrac{j}{-1} = -j$

b)

MAGNITUDE $(z) = |z| =$ length of vector

IF $|z| = 2$, THEN CIRCLE OF RADIUS 2 AROUND ORGIN

IF $|z - j| = 2$, THEN CIRCLE BUT OFFSET BY j

2.36

$Im(z) = \pi$
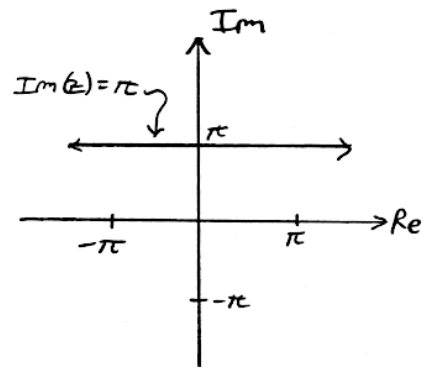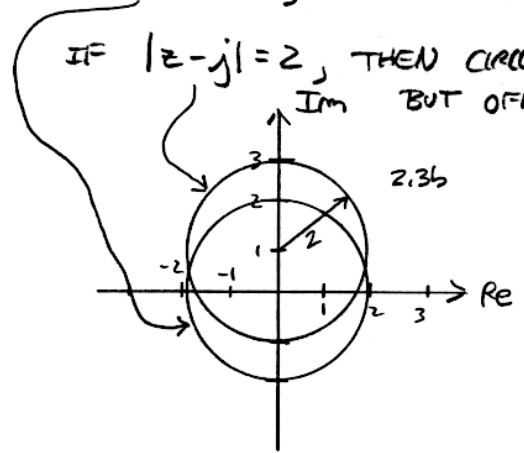
c) $Im(z) = \pi$ THE REAL PART OF z IS ANYTHING