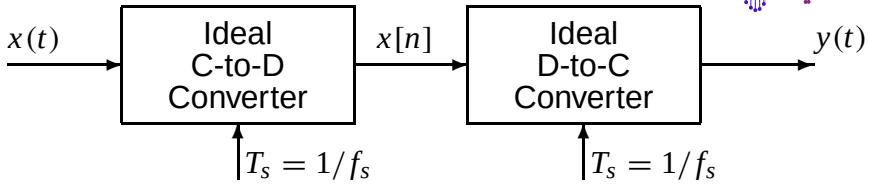
PROBLEM:



(a) If the input to the ideal C/D converter is a sinusoid with frequency of 700 Hz, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the frequency of the output.

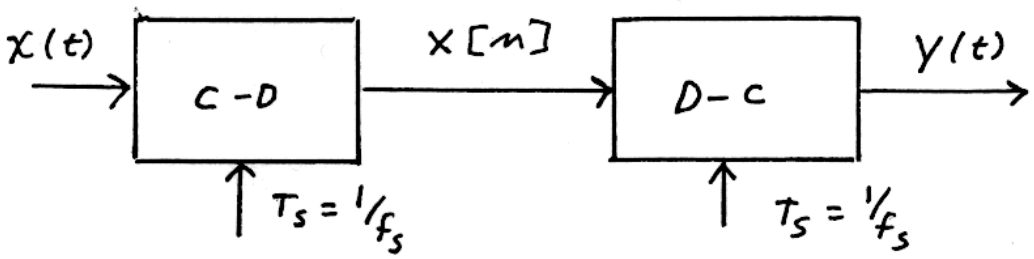
(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



$$x(t) = A\cos 2\pi 700t + \phi$$

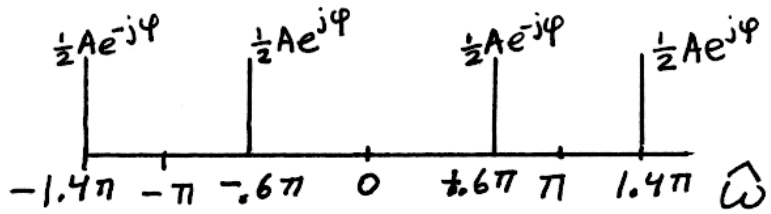
$$x[n] = x(t)\Big|_{t=nT_s}$$

$$T_s = \frac{1}{f_s} = \frac{1}{1000}$$

$$x[n] = A\cos(1.4\pi n + \phi)$$

we now reconstruct with $f_s = 1000$

$$x[n] = A\cos(\hat{\omega} n + \phi) \qquad \hat{\omega} = 1.4\pi$$



the $\hat{\omega}$ components AT $\pm 1.4\pi$
Alias (fold) To components AT $\mp .6\pi$

$$y(t) = A\cos(.6\pi n - \phi)\Big|_{n \leftarrow f_s t}$$

$$\text{frequency} = \frac{.6}{2T_s} = .3f_s = 300$$

Note $1000 - 700 = 300$.

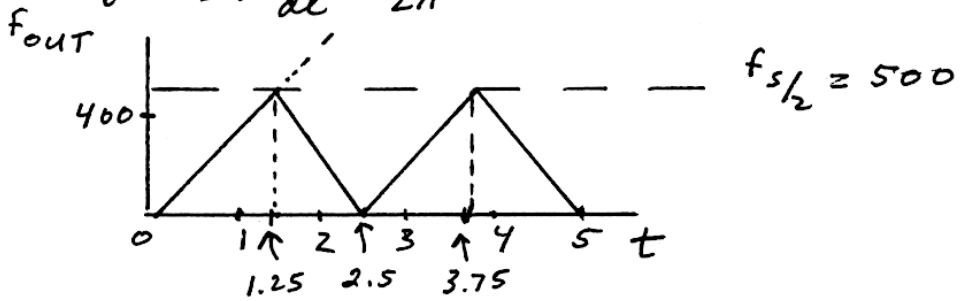
$$y(t) = A\cos(2\pi(300)t - \varphi)$$

(b) $x(t) = \cos 400\pi t^2 \qquad 0 \le t \le 5$

INSTANTANEOUS frequency

$x(t) = A \cos \psi(t)$

$f_i = \frac{1}{2\pi}\frac{d\psi}{dt} = \frac{1}{2\pi}(800\pi t) = 400t$



foloing occurs AT $f_i = 500, 1500$

reconstructor Aliases (CANNOT reconstruct)

Above $f_{s/2} = 500$ Hz.

Note: To COMPARE TO PART (a) 700 INPUT

occurs AT $t = 7/4 = 1.75$ sec.

EQUATION of Line in figure for $1.25 \le t \le 2.5$

is $f_{OUT} = -400t + 1000$

at $t = 7/4$

$f_{OUT} = -400(7/4) + 1000 = 300$, agrees with part a