PROBLEM:

A linear time-invariant system has system function

$$H(z) = (1 - z^{-2})(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4})$$

(a) Determine the impulse response of this system.

(b) The input to this system is

$$x[n] = 3 + 7\delta[n] + 10\cos(\pi n/2)$$

Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n. (*Note: This is an easy problem if you approach it correctly!*)

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SOLUTION

a) $H(z) = (1-z^{-2})(1+z^{-1}+z^{-2}+z^{-3}+z^{-4})$

$$= \begin{pmatrix} 1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+ \\ -z^{-2}-z^{-3}-z^{-4}-z^{-5}-z^{-6} \end{pmatrix}$$

$$= 1+z^{-1}+0+0+0-z^{-5}-z^{-6} = \frac{Y(z)}{X(z)} = H(z)$$

$$h[n] = \{1, 1, 0, 0, 0, -1, -1\}$$

(b) Use Linearity to find the response to each input separately and then add them together:

$x_1[n] = 3$

use the frequency response at $\hat{\omega}=0$.

$$H(e^{j0}) = H(z)\big|_{z=1} = (1-1)(1+1+1+1+1) = 0$$

Thus $y_1[n] = 0$

$x_2[n] = 7\delta[n] \implies y_2[n] = 7\,h[n]$

$x_3[n] = 10\cos(n\pi/2)$

Use freq. response at $\hat{\omega} = \pi/2$

$$H(e^{j\pi/2}) = H(z)\big|_{z=e^{j\pi/2}=j}$$

$$= (1-j^{-2})(1+j^{-1}+j^{-2}+j^{-3}+j^{-4})$$

$$= (1+1)(1-j-1+j+1) = 2$$

$y_3[n] = 20\cos(\frac{\pi}{2}n)$

$y[n] = y_1[n] + y_2[n] + y_3[n]$

$$= 7\delta[n] + 7\delta[n-1] - 7\delta[n-5] - 7\delta[n-6] + 20\cos(\tfrac{\pi}{2}n)$$

(b) ALTERNATE solution in the time domain:

$$y[n] = x[n] + x[n-1] + 0 + 0 + 0 - x[n-5] - x[n-6]$$

For $x[n] = 3 + 7\,\delta[n] + 10\cos\left(n\frac{\pi}{2}\right)$

$$\begin{array}{lll}
y[n] = & 3 + 7\delta[n] + \left\{10\cos\left(\frac{\pi}{2}n\right) = 10\cos\left(\frac{\pi}{2}n - \left[0\frac{\pi}{2} = 0\right]\right)\right\} + & n \\
 & 3 + 7\delta[n-1] + \left\{10\cos\left(\frac{\pi}{2}(n-1)\right) = 10\cos\left(\frac{\pi}{2}n - \left[\frac{\pi}{2} = \frac{\pi}{2}\right]\right)\right\} + & n-1 \\
 & -3 - 7\delta[n-5] - \left\{10\cos\left(\frac{\pi}{2}(n-5)\right) = 10\cos\left(\frac{\pi}{2}n - \left[\frac{5\pi}{2} = \frac{\pi}{2}\right]\right)\right\} + & n-5 \\
 & -3 - 7\delta[n-6] - \left\{10\cos\left(\frac{\pi}{2}(n-6)\right) = 10\cos\left(\frac{\pi}{2}n - \left[\frac{6\pi}{2} = 3\pi\right]\right)\right\} & n-6
\end{array}$$

terms: n, $n-1$, $n-5$, $n-6$

cancel

Terms for n & $n-6$ will add; terms at $n-1$ & $n-5$ cancel

$$20\cos\left(\frac{\pi}{2}n\right)$$

$$y[n] = 7\left(\delta[n] + \delta[n-1] - \delta[n-5] - \delta[n-6]\right) + 20\cos\left(\frac{\pi}{2}n\right)$$

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