PROBLEM:

For each of the difference equations below, determine the poles and zeros of the corresponding system function, $H(z)$.

$$\mathcal{S}_1: \qquad y[n] = 0.4y[n-1] + x[n] + x[n-1]$$

$$\mathcal{S}_3: \qquad y[n] = -0.75y[n-1] + x[n] - x[n-1]$$

$$\mathcal{S}_6: \qquad y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$$

$$\mathcal{S}_7: \qquad y[n] = x[n] + \tfrac{1}{4}x[n-1] - \tfrac{3}{4}x[n-2]$$

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SOLUTION

POLES + ZEROS

Z-DOMAIN

(S1) $y[n] = 0.4\,y[n-1] + x[n] + x[n-1]$

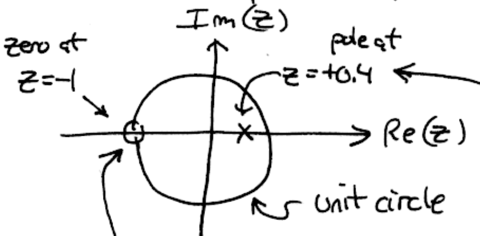
$Y(z)\left[1 - 0.4z^{-1}\right] = X(z)\left[1 + z^{-1}\right]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\text{Output}}{\text{Input}} = \frac{(1+z^{-1})}{(1-0.4z^{-1})}$$

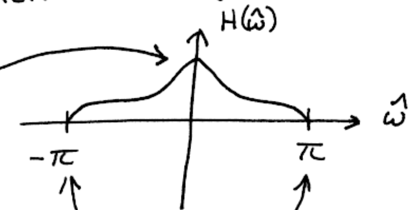
← "zeros" = roots of numerator ⇒ $1 + z^{-1} = 0$ when $z = -1$ → $H(z = \text{a "zero"}) = 0$

← "poles" = roots of denominator ⇒ $1 - 0.4z^{-1} = 0$ when $z = 0.4$ → $H(z = \text{a "pole"}) \rightarrow \infty$

Illustrate on complex z-plane:



Relate to $\hat{\omega}$ ($|H(\hat{\omega})|$ vs $\hat{\omega}$)



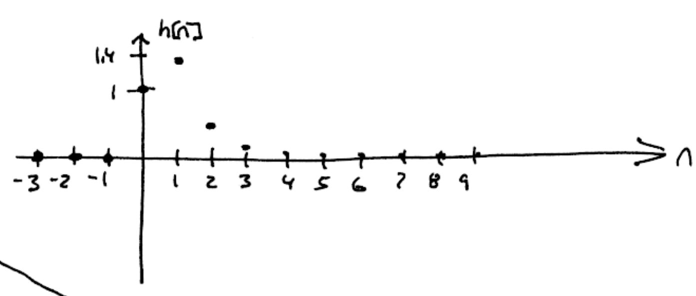
Higher "Gain" due to pole near "DC"

$|H(\hat{\omega} = \pi)| = 0$ = "Notch" or "Null" at $\hat{\omega} = \pi$ or $\frac{1}{2} f_s$

FIND IMPULSE RESPONSE = $h[n]$

$$h[n] = y[n]\Big|_{x[n]=\delta[n]} = \{1,\ 1+0.4,\ 0.4(1+0.4),\ 0.4^2(1+0.4),\ 0.4^n(1+0.4)\ldots\}$$

$$= \begin{cases} 0 & n<0 \\ 1 & n=0 \\ 0.4^{n-1}(1.4) & n>0 \end{cases}$$



ANALYTIC FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z = e^{j\hat{\omega}})$$

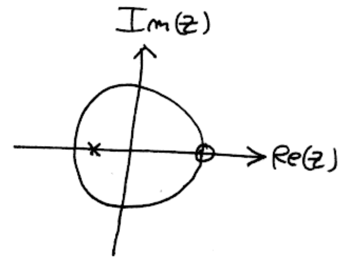
$$= \frac{1 + e^{-j\omega}}{1 - 0.4e}$$

DC Gain = $H(\hat{\omega} = 0)$ =

$$= \frac{1 + e^{j0}}{1 - 0.4e^{j0}} = \frac{1+1}{1-0.4} = \frac{2}{0.6} = 3.333\ldots$$

(53) $y[n] = -0.75y[n-1] + x[n] - x[n-1]$

$H(z) = \dfrac{(1-z^{-1})}{(1+0.75z^{-1})}$ $\rightarrow$ zero at $z=1$, $\rightarrow$ pole at $z=-0.75$



$$H(\hat{\omega}) = \frac{1-e^{-j\hat{\omega}}}{1+0.75e^{-j\hat{\omega}}}$$

$$h[n] = \{1, -1-0.75, -0.75(-1-0.75), (-0.75)^2(-1-0.75) \ldots\}$$

$$= \begin{cases} 0 & n<0 \\ 1 & n=0 \\ (-0.75)^{n-1}(-1.75) & n>0 \end{cases}$$

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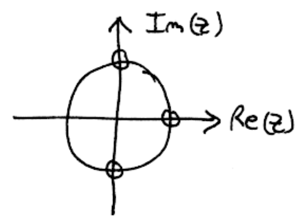
(56) $y[n] = x[n] - x[n-1] + x[n-2] - x[n-3]$

$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} =$

zero roots at $z = 1, \pm j = 1, e^{\pm j\frac{\pi}{2}}$



$h[n] = \{1, -1, 1, -1\}$

(no poles $\Leftrightarrow$ FIR only)

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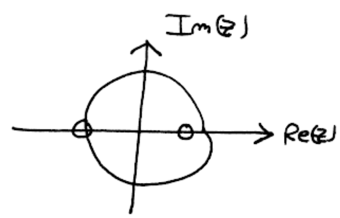
(57) $y[n] = x[n] + \frac{1}{4}x[n-1] - \frac{3}{4}x[n-2]$

$H(z) = 1 + \frac{1}{4}z^{-1} - \frac{3}{4}z^{-2} = (1+z^{-1})(1-\frac{3}{4}z^{-1})$

zero roots at $z = -1, \frac{3}{4}$

no poles ($\Leftrightarrow$ FIR only)



$h[n] = \{1, \frac{1}{4}, -\frac{3}{4}\}$ $\rightarrow$ PLOT 7.5L

$H(\hat{\omega}) = (1+e^{-j\hat{\omega}})(1-\frac{3}{4}e^{-j\hat{\omega}})$

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