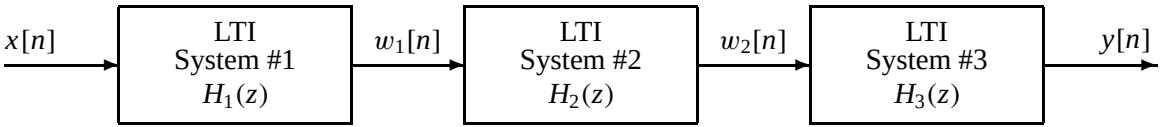
PROBLEM:

A cascade of three FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 + z^{-1})(1 - 0.2z^{-1}) \quad \text{and} \quad h_2[n] = (0.8)^n u[n] \quad \text{and} \quad h_3[n] = (0.2)^{n-1} u[n-1]$$

(a) Determine a *simplified* system function $H(z)$ for the overall system.

$H(z) =$

(b) Determine the impulse response for the overall system.

$h[n] =$

(c) For the input $x[n] = \delta[n] - 0.8\delta[n-1]$, determine the output $y[n]$.

$y[n] =$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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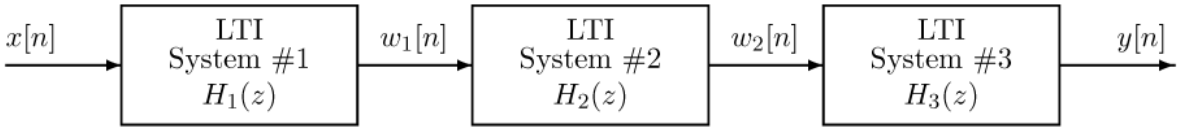
SOLUTION

A cascade of three FIR discrete-time systems is depicted by the following block diagram:



The systems are defined by the following:

$$H_1(z) = (1 + z^{-1})(1 - 0.2z^{-1}) \quad \text{and} \quad h_2[n] = (0.8)^n u[n] \quad \text{and} \quad h_3[n] = (0.2)^{n-1} u[n-1]$$

(a) Determine a *simplified* system function $H(z)$ for the overall system.

$$H(z) = (1 + z^{-1})(1 - 0.2z^{-1}) \tfrac{1}{1-0.8z^{-1}} \tfrac{z^{-1}}{1-0.2z^{-1}} = \tfrac{z^{-1}+z^{-2}}{1-0.8z^{-1}} = \tfrac{z^{-1}}{1-0.8z^{-1}} + \tfrac{z^{-2}}{1-0.8z^{-1}}$$

$$\boxed{H(z) = \tfrac{z^{-1}}{1-0.8z^{-1}} + \tfrac{z^{-2}}{1-0.8z^{-1}}}$$

(b) Determine the impulse response for the overall system.

$H(z) = \tfrac{z^{-1}}{1-0.8z^{-1}} + \tfrac{z^{-2}}{1-0.8z^{-1}}$

$$h[n] = 0.8^{n-1}u[n-1] + 0.8^{n-2}u[n-2] = 0.8^n u[n] * \{\delta[n-1] + \delta[n-2]\}$$

$$\boxed{h[n] = 0.8^{n-1}u[n-1] + 0.8^{n-2}u[n-2] = 0.8^n u[n] * \{\delta[n-1] + \delta[n-2]\}}$$

(c) For the input $x[n] = \delta[n] - 0.8\delta[n-1]$, determine the output $y[n]$.

$X(z) = 1 - 0.8z^{-1}$
$Y(z) = X(z)H(z) = (1 - 0.8z^{-1})(\tfrac{z^{-1}}{1-0.8z^{-1}} + \tfrac{z^{-2}}{1-0.8z^{-1}}) = z^{-1} + z^{-2}$
$y[n] = \delta[n-1] + \delta[n-2]$

or alternatively

$$\begin{aligned} y[n] &= (\delta[n] - 0.8\delta[n-1]) * (0.8^n u[n]) * (\delta[n-1] + \delta[n-2]) \\ &= (0.8^n u[n] - 0.8(0.8)^{n-1} u[n-1]) * (\delta[n-1] + \delta[n-2]) \\ &= (1) * (\delta[n-1] + \delta[n-2]) = \delta[n-1] + \delta[n-2] \end{aligned}$$

$$\boxed{y[n] = \delta[n-1] + \delta[n-2]}$$

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