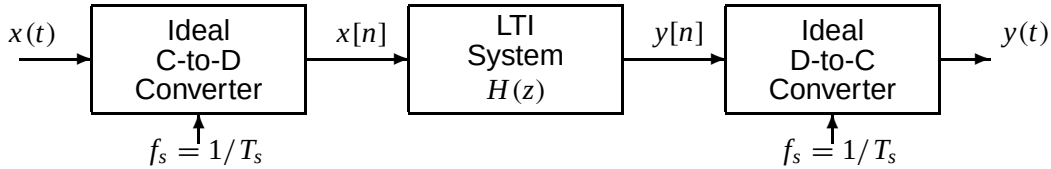
PROBLEM:

Consider the following system for sampling, filtering, and reconstruction of a continuous-time signal:



where $f_s = 1000$ samples/sec, the LTI system has the system function $H(z) = 3z^{-2}$, and the continuous-time input signal is

$$x(t) = 2\cos(750\pi t + \pi/4) + 3\cos(1600\pi t - 3\pi/5).$$

(a) Plot the complete frequency spectrum for $x[n]$ in the region $-\pi < \hat{\omega} \leq \pi$.

(b) Determine an expression for the output $y(t)$ of this system for the input $x(t)$ indicated.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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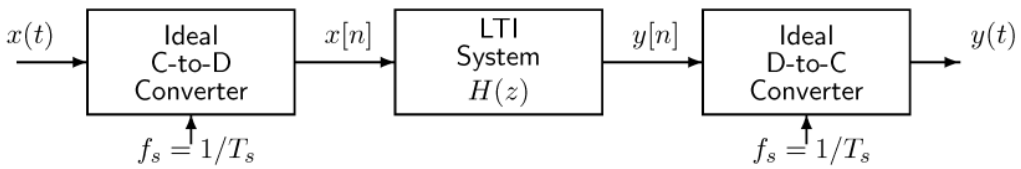
SOLUTION

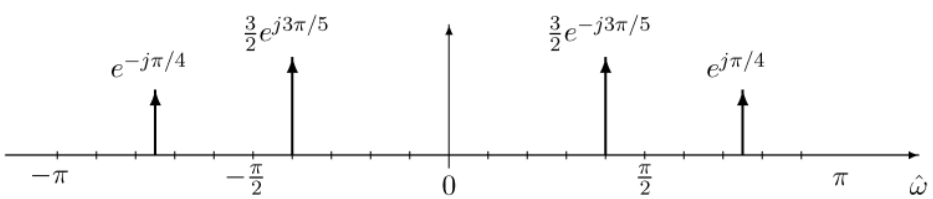
Consider the following system for sampling, filtering, and reconstruction of a continuous-time signal:



where $f_s = 1000$ samples/sec, the LTI system has the system function $H(z) = 3z^{-2}$, and the continuous-time input signal is

$$x(t) = 2\cos(750\pi t + \pi/4) + 3\cos(1600\pi t - 3\pi/5).$$

(a) Plot the complete frequency spectrum for $x[n]$ in the region $-\pi < \hat{\omega} \leq \pi$.



(b) Determine an expression for the output $y(t)$ of this system for the given input $x(t)$ indicated above.

$$\begin{aligned} x[n] &= x(n/f_s) = 2\cos(750\pi\tfrac{n}{1000} + \pi/4) + 3\cos(1600\pi\tfrac{n}{1000} - 3\pi/5) \\ &= 2\cos(0.75\pi n + \pi/4) + 3\cos(1.6\pi n - 3\pi/5) \\ &= 2\cos(0.75\pi n + \pi/4) + 3\cos(-0.4\pi n - 3\pi/5) \\ &= 2\cos(0.75\pi n + \pi/4) + 3\cos(0.4\pi n + 3\pi/5) \end{aligned}$$

$$\begin{aligned} y[n] &= 3x[n-2] = 3(2\cos(0.75\pi(n-2) + \pi/4) + 3\cos(0.4\pi(n-2) + 3\pi/5)) \\ &= 6\cos(0.75\pi n - 1.5\pi + \pi/4) + 9\cos(0.4\pi n - 0.8\pi + 3\pi/5) \\ &= 6\cos(0.75\pi n + 3\pi/4) + 9\cos(0.4\pi n - 8\pi/10 + 6\pi/10) \\ &= 6\cos(0.75\pi n + 3\pi/4) + 9\cos(0.4\pi n - \pi/5) \end{aligned}$$

$$y(t) = y[tf_s] = 6\cos(0.75\pi t(1000) + 3\pi/4) + 9\cos(0.4\pi t(1000) - \pi/5)$$

$$y(t) = 6\cos(750\pi t + 3\pi/4) + 9\cos(400\pi t - \pi/5)$$

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