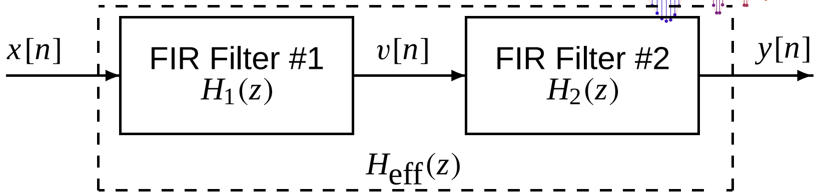
PROBLEM:



Consider the above LTI system where

$$v[n] = x[n] + 0.5x[n-2] \qquad \text{and} \qquad H_2(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

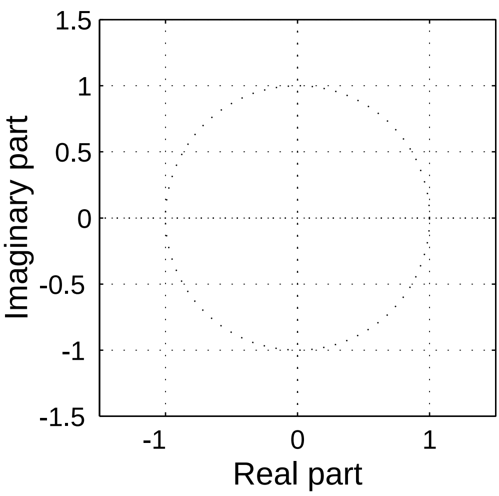
(a) Determine the system functions $H_1(z)$ and $H_2(z)$.

$H_1(z) =$

$H_2(z) =$

(b) Determine the difference equation that relates the output $y[n]$ to the input $x[n]$.

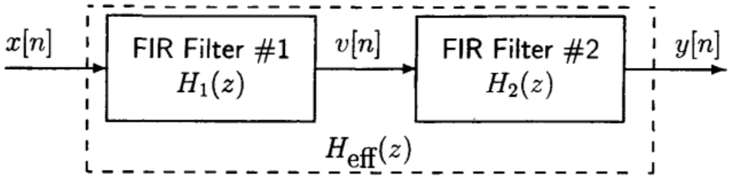
(c) Determine *all* of the zeros of the equivalent system $H_{\text{eff}}(z)$ and plot them in the z-plane.



(d) Determine the input frequencies $\hat{\omega}$ that are nulled by this system (assume no aliasing). **EXPLAIN your answer.**

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION



Consider the above LTI system where

$$v[n] = x[n] + 0.5x[n-2] \qquad \text{and} \qquad H_2(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

(a) Determine the system functions $H_1(z)$ and $H_2(z)$.

$$V(z) = X(z) + \tfrac{1}{2} z^{-2} X(z)$$

$$z = e^{j\hat{\omega}}$$

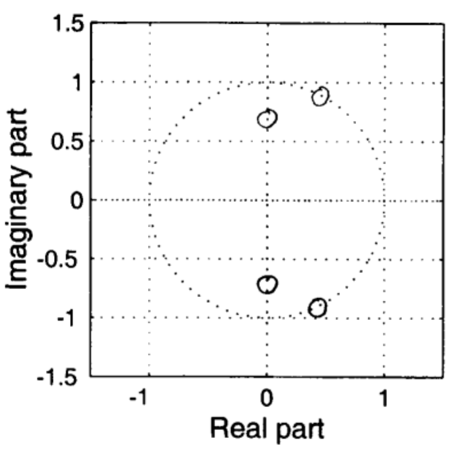
$\boxed{H_1(z) = 1 + \tfrac{1}{2} z^{-2}}$ $\boxed{H_2(z) = 1 - z^{-1} + z^{-2}}$

(b) Determine the difference equation that relates the output $y[n]$ to the input $x[n]$.

$$H_{\text{eff}}(z) = H_1(z) H_2(z) = \left(1 + \tfrac{1}{2} z^{-2}\right)\left(1 - z^{-1} + z^{-2}\right)$$
$$= 1 - z^{-1} + z^{-2} + \tfrac{1}{2} z^{-2} - \tfrac{1}{2} z^{-3} + \tfrac{1}{2} z^{-4}$$
$$= 1 - z^{-1} + \tfrac{3}{2} z^{-2} - \tfrac{1}{2} z^{-3} + \tfrac{1}{2} z^{-4}$$

$$y[n] = x[n] - x[n-1] + \tfrac{3}{2} x[n-2] - \tfrac{1}{2} x[n-3] + \tfrac{1}{2} x[n-4]$$

(c) Determine *all* of the zeros of the equivalent system $H_{\text{eff}}(z)$ and plot them in the z-plane.



Zeros for $H_1(z)$

$$\frac{0 \pm \sqrt{0 - 4(\frac{1}{2})}}{2} = \pm j \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} e^{\pm j \frac{\pi}{2}}$$

Zeros for $H_2(z)$

$$\frac{+1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm j \frac{\sqrt{3}}{2} = e^{\pm j \frac{\pi}{3}}$$

(d) Determine the input frequencies $\hat{\omega}$ that are nulled by this system (assume no aliasing).

Zeros on unit circle will null.

Hence, $\hat{\omega} = \left\{ \pm \frac{\pi}{3} \right\}$ will be nulled.