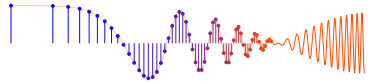

PROBLEM:

For the same system as in the previous problem:

$$\hat{H}(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$$

(a) This system can "block" certain input signals. For which input frequencies $\omega_\circ$, is the response to $x[n] = \cos(\omega_\circ n)$ equal to zero?

(b) When the input to the system is $x[n] = \cos(\pi n/3)$ determine the output signal $y[n]$ in the form:

$$A\cos(\omega_\circ n + \phi)$$

Give numerical values for the constants A, $\omega_\circ$ and ϕ.

SOLUTION

For the same system as in the previous problem:

$$H(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$$

(a) This system can "block" certain input signals. For which input frequencies $\hat{\omega}_o$, is the response to $x[n] = \cos(\hat{\omega}_o n)$ equal to zero?

Zeros of $H(z)$ at $z = 1, -1, j, -j = e^{j0}, e^{j\pi}, e^{j\pi/2}, e^{-j\pi/2}$

$\therefore$ BLOCK $\hat{\omega} = 0, \pi$ & $\pi/2$

because $\cos\frac{\pi n}{2} = \frac{1}{2}e^{j\pi n/2} + \frac{1}{2}e^{-j\pi n/2}$

(b) When the input to the system is $x[n] = \cos(\pi n/3)$ determine the output signal $y[n]$ in the form:

$$A\cos(\hat{\omega}_o n + \phi)$$

Give numerical values for the constants A, $\hat{\omega}_o$ and ϕ.

Find mag & phase at $\hat{\omega} = \pi/3$

$$M(\pi/3) = 2\sin 2\pi/3 = 2\frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\varphi(\pi/3) = \pi/2 - 2(\pi/3) = \frac{\pi}{2} - \frac{2\pi}{3} = \frac{3\pi - 4\pi}{6} = -\pi/6$$

$\therefore$ $y[n] = \sqrt{3}\cos\left(\frac{\pi n}{3} - \frac{\pi}{6}\right)$

OR:

$$y[n] = x[n] - x[n-4] = \cos\frac{\pi n}{3} - \cos\left(\frac{\pi(n-4)}{3}\right).$$

$$= \cos\frac{\pi n}{3} - \cos\left(\frac{\pi n}{3} - \frac{4\pi}{3}\right)$$

$$= \cos\frac{\pi n}{3} + \cos\left(\frac{\pi n}{3} - \frac{\pi}{3}\right)$$

$$= \sqrt{3}\cos\left(\frac{\pi n}{3} - \frac{\pi}{6}\right).$$

$(1 + \frac{1}{2}) - j\frac{\sqrt{3}}{2}$

$\frac{3}{2} - j\frac{\sqrt{3}}{2}$

$\sqrt{3}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$