PROBLEM:

Let $x[n]$ be the complex exponential

$$x[n] = e^{j(0.3\pi n - 0.2\pi)}$$

If we define a new signal $y[n]$ to be the result of processing $x[n]$ through a system whose z-transform operator is $\hat{H}(z) = 1 + z^{-10}$, it is possible to express $y[n]$ in the form

$$y[n] = Ae^{j(\omega_0 n + \phi)} \tag{1}$$

(a) Draw a (rotating) phasor diagram to illustrate how $y[n]$ is formed from $x[n]$.

(b) Determine the numerical values of A, ϕ and ω_0 in (1).

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

Let $x[n]$ be the complex exponential

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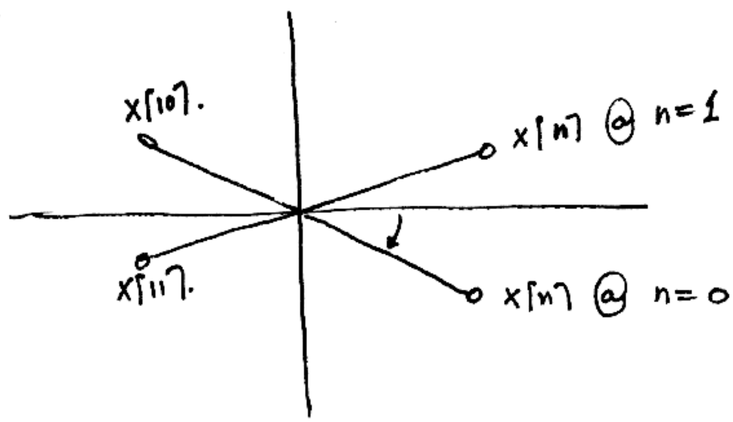
If we define a new signal $y[n]$ to be the result of processing $x[n]$ through a system whose z-transform operator is $H(z) = 1 + z^{-10}$, it is possible to express $y[n]$ in the form

$$y[n] = Ae^{j(\hat{\omega}_0 n + \phi)} \tag{1}$$

(a) Draw a (rotating) phasor diagram to illustrate how $y[n]$ is formed from $x[n]$.

$$x[n] \longrightarrow \boxed{1+z^{-10}} \longrightarrow y[n] = x[n] + x[n-10].$$

$$x[10] = e^{j\,0.3\pi(10) - 0.2\pi}$$
$$= -e^{-j0.2\pi}$$



$\therefore\ y[10] = 0$

(b) Determine the numerical values of A, ϕ and $\hat{\omega}_0$ in (1).

$A = 0$

$\hat{\omega}_0 = 0.3\pi$

$\varphi =$ anything.

$$H(e^{j\hat{\omega}}) = 1 + e^{-j10\hat{\omega}}$$

$$H(e^{j\hat{\omega}})\Big|_{\omega = 0.3\pi} = 1 + e^{-j3\pi} = 1 - 1 = 0$$

$$y[n] = H(e^{j\hat{\omega}})\Big|_{\omega=0.3\pi} \times x[n]$$

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