



PROBLEM:

Suppose that a system is defined by the following operator

$$\hat{H}(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$$

- Write the time-domain description of this system—in the form of a difference equation.
- Write the formula for the frequency response of the system.
- Derive simple formulas for the magnitude response versus ω , and the phase response versus ω . These formulas must contain no complex terms and no square roots.



Suppose that a system is defined by the following operator

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(a) Write the time-domain description of this system—in the form of a difference equation.

$$(1 - z^{-1})(1 + z^{-1})(1 + z^{-2}) = (1 - z^{-2})(1 + z^{-2}) = 1 - z^{-4}$$

$$H(z) = 1 - z^{-4}$$

$$\Rightarrow y[n] = x[n] - x[n-4]$$

(b) Write the formula for the frequency response of the system.

$$H(e^{j\hat{\omega}}) = 1 - e^{-j4\hat{\omega}}$$

(c) Derive simple formulas for the magnitude response versus $\hat{\omega}$, and the phase response versus $\hat{\omega}$. These formulas must contain no complex terms and no square roots.

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} \frac{(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})}{2j}$$

$$= 2j e^{-j2\hat{\omega}} \sin 2\hat{\omega}$$

$$= e^{j\pi/2} e^{-j2\hat{\omega}} 2 \sin 2\hat{\omega}$$

$$M(\hat{\omega}) = 2 \sin 2\hat{\omega}$$

$$\varphi(\hat{\omega}) = -2\hat{\omega} + \pi/2$$