PROBLEM:

A discrete-time signal $x[n]$ is known to be a sinusoid:

$$x[n] = A\cos(\omega_0 n + \phi)$$

The values of $x[n]$ are tabulated for $n = 0, 1, 2, 3, 4,$ and 5.

n	0	1	2	3	4	5
$x[n]$	1.5708	0.2348	-1.1478	-2.3030	-3.0020	-3.1065

Plot $x[n]$, and then determine the numerical values of A, ϕ and ω_0. (This is not an easy calculation.)

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Data was generated in MATLAB via:

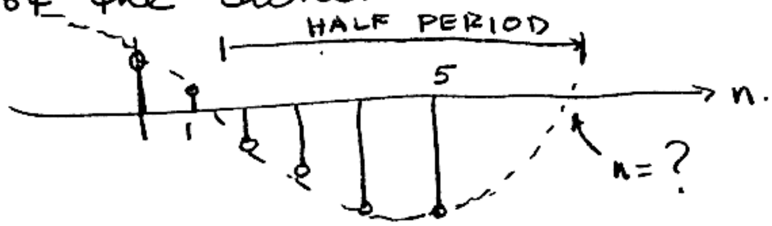
pi * cos (pi/7 * n + pi/3).

So the <u>ANSWER</u> is: $A = \pi$, $\omega_0 = \pi/7$, $\varphi = \pi/3$

How do you find it?

$\boxed{\text{Method 1}}$ (Trial and error).

plot the data. —— and then guess



Imagine an envelope:

Guess $A = 3.2$ because it has to be greater than 3.1065

Guess period $\approx 7 \times 2 \Rightarrow \omega_0 = \frac{2\pi}{14} = \pi/7$.

Guess phase $= \cos^{-1}\left(\frac{1.57}{3.2}\right) = 1.058 \rightarrow 60.6^\circ$

After making the guesses, synthesize

$A\cos(\omega_0 n + \varphi)$

and compare to given data. Plot the error and adjust the values of A, ω_0, φ.

$\boxed{\text{METHOD 2}}$

Use Algebra, trig, or complex numbers.

Here's a trick:

write $x[n]$ as $\Re e\{\underbrace{Ae^{j\varphi}e^{j\omega_0 n}}_{\text{call this } z[n]}\}$.

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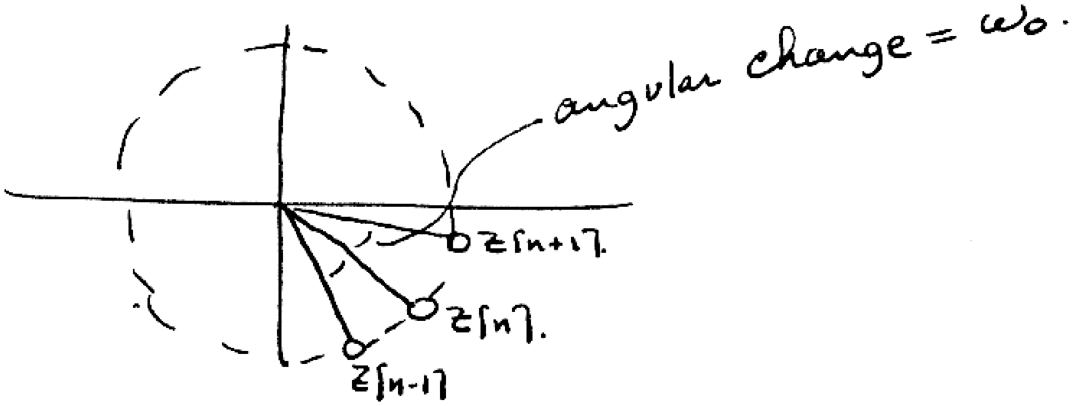
Prob (cont).

write out 3 consecutive $z[n]$'s

$$z[n-1] = Ae^{j\varphi}e^{j\omega_0(n-1)}$$ — rotate by $e^{-j\omega_0}$

$$z[n] = Ae^{j\varphi}e^{j\omega_0 n}$$ — rotate by $e^{+j\omega_0}$

$$z[n+1] = Ae^{j\varphi}e^{j\omega_0(n+1)}.$$



Now you can evaluate:

$$\frac{z[n+1]}{z[n]} + \frac{z[n-1]}{z[n]} = ?$$

$$e^{+j\omega_0} + e^{-j\omega_0} = 2\cos\omega_0$$

This gives a method to find ω_0.

$$(2\cos\omega_0)\,z[n] = z[n+1] + z[n-1].$$

Take the real part of this eqn

$$(2\cos\omega_0)\,x[n] = x[n+1] + x[n-1].$$

$$\Rightarrow \cos\omega_0 = \frac{x[n+1] + x[n-1]}{2x[n]}.$$

Prob (cont)

So we can calculate w_0

$$w_0 = \cos^{-1}\left(\frac{x[2]+x[0]}{2x[1]}\right) = \cos^{-1}\left(\frac{-1.1478+1.5708}{2(0.2348)}\right).$$

$$= \cos^{-1}(0.9010) = 0.4488 = \pi/7.$$

Once you find w_0 the rest is Relatively EASY:

$$\left.\begin{aligned} x[1] &= A\cos(w_0+\varphi) \\ x[0] &= A\cos\varphi \end{aligned}\right] \Rightarrow \frac{x[1]}{x[0]} = 0.1495 = \frac{\cos(w_0+\varphi)}{\cos\varphi}.$$

$$\Rightarrow 0.1495 = \frac{\cos w_0 \cos\varphi - \sin w_0 \sin\varphi}{\cos\varphi} = \underset{0.9010}{\cos w_0} - \underset{0.4339.}{\sin w_0}\tan\varphi.$$

$$\Rightarrow \tan\varphi = \frac{0.1495-0.9010}{-0.4339} = 1.7321 \Rightarrow \varphi = \pi/3.$$

Finally $A = x[0]/\cos\varphi. = 1.5708/\tfrac{1}{2}$

$$= 3.14159\ldots = \pi.$$

WHEW! NON-LINEAR EQNS are hard to SOLVE, BUT the formula

$$\cos w_0 = \frac{x[n+1]+x[n-1]}{2x[n]}$$

turns out to be important.