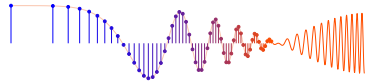


PROBLEM:

Suppose that a system is defined by the following operator

$$H(z) = 1 - z^{-1}$$

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) Write the formula for the frequency response of the system.

(c) Sketch a plot of the magnitude response versus ω.

(d) When the input to the system is $x[n] = \sin(\pi n/100)$ determine the functional form for the output signal $y[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$H(z) = 1 - z^{-1}$ (DELAY)

(a) $y[n] = x[n] - x[n-1]$.

(b) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = \boxed{1 - e^{-j\hat{\omega}}}$

(c) Formula for magnitude

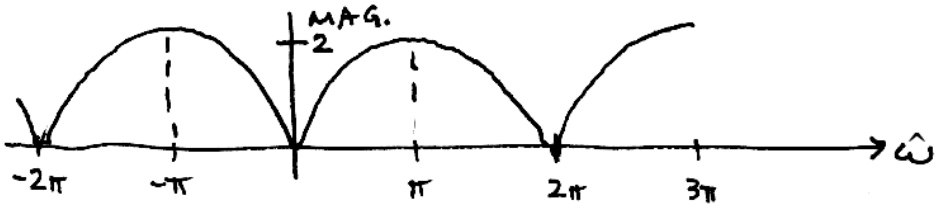
$$|H(e^{j\hat{\omega}})| = \sqrt{(1-\cos\hat{\omega})^2 + \sin^2\hat{\omega}}$$

or

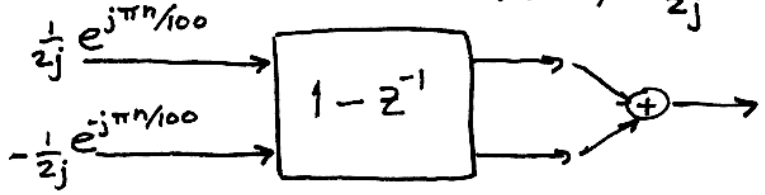
$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} = e^{-j\hat{\omega}/2}\left(e^{j\hat{\omega}/2} - e^{-j\hat{\omega}/2}\right)$$

$$= 2j\, e^{-j\hat{\omega}/2} \sin\hat{\omega}/2$$

$$\Rightarrow |H(e^{j\hat{\omega}})| = 2\,|\sin\hat{\omega}/2|$$



(d) When $x[n] = \sin\left(\frac{\pi n}{100}\right) = \frac{1}{2j}e^{+j\frac{\pi n}{100}} - \frac{1}{2j}e^{-j\frac{\pi n}{100}}$



Find the responses separately & add.

Response due to $\frac{1}{2j}e^{+j\pi n/100}$ is

$$H(e^{j\pi/100}) \cdot \frac{1}{2j} e^{j\pi n/100}$$

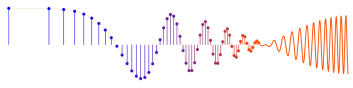
EVALUATE: $H(e^{j\pi/100}) = 2j\, e^{-j\pi/200} \sin\frac{\pi}{200}$

COMBINE; $2j$ CANCELS

$$\frac{\pi}{200} e^{-j\pi/200} e^{j\pi n/100}$$

LEAVE IN THIS FORM FOR LATER

$\sin\frac{\pi}{200} \approx \pi/200$ because $\sin\epsilon \approx \epsilon$



PROB (cont)

Response due to $\frac{-1}{2j} e^{-j\pi n/100}$ is:

$$H(e^{-j\pi/100}) \cdot \frac{-1}{2j} e^{-j\pi n/100} = \frac{\pi}{200} e^{+j\pi/200} e^{-j\pi n/100}$$

COMBINE

$$H(e^{-j\pi/100}) = 2j e^{+j\pi/200} \sin\left(\frac{-\pi}{200}\right) \approx 2j\left(\frac{-\pi}{200}\right) e^{+j\pi/200}$$

Add these two together:

$$y[n] = \frac{\pi}{200} e^{j(\pi n/100 - \pi/200)} + \frac{\pi}{200} e^{-j(\pi n/100 - \pi/200)}$$

$$y[n] = \frac{\pi}{200} \cdot 2\cos\left(\frac{\pi n}{100} - \frac{\pi}{200}\right).$$

$$= \frac{\pi}{100} \cos\left(\frac{\pi n}{100} - \frac{\pi}{200}\right).$$

MAG CHANGE

PHASE CHANGE

NOTICE

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\frac{\pi}{100}} = 2j e^{-j\pi/200} \sin\frac{\pi}{200}$$

$$= 2\left(\frac{\pi}{200}\right) e^{j(\pi/2 - \pi/200)}.$$

$$= \frac{\pi}{100} e^{j(\pi/2 - \pi/200)}$$

compare

COMPARE

MAG

PHASE