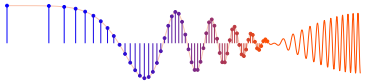

PROBLEM:

Suppose that a system is defined by the following operator

$$H(z) = 1 - z^{-1}$$

(a) Prove that when the input to this system is periodic with period M, that the output will also be periodic with the same period.

(b) When the input to the system is the periodic signal:

$$x[n] = \begin{cases} +1 & \text{when } n = 0, \pm 3, \pm 6, \ldots \\ -1 & \text{when } n = \ldots, -8, -5, -2, 1, 4, 7, \ldots \\ 0 & \text{when } n = \ldots, -7, -4, -1, 2, 5, 8, \ldots \end{cases}$$

determine the functional form for the output signal $y[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$H(z) = 1 - z^{-1}$$

$$\therefore \quad y[n] = x[n] - x[n-1].$$

(a) $x[n]$ is periodic means $x[n+M] = x[n]$ $\forall n$.

Look at $y[n+M] = x[n+M] - x[n+M-1]$

↓ TRUE BECAUSE $x[\cdot]$ IS PERIODIC

$y[n] \leftarrow x[n] - x[n-1]$.

$\therefore$ this is $y[n]$

$\therefore$ $y[n] = y[n+M]$ which means $y[n]$ is periodic with period $= M$.

(b) $$x[n] = \begin{cases} +1 & \text{when } n = 0, \pm 3, \pm 6, \ldots \\ -1 & \text{when } n = \ldots -5, -2, 1, 4, 7, \ldots \\ 0 & \text{when } n = \ldots -4, -1, 2, 5, 8 \ldots \end{cases}$$

$$y[0] = x[0] - x[-1] = 1 - (0) = 1$$

$$y[1] = x[1] - x[0] = -1 - (1) = -2$$

$$y[2] = x[2] - x[1] = 0 - (-1) = 1$$

$$y[3] = x[3] - x[2] = +1 - 0 = 1$$

$$y[4] = x[4] - x[3] = -1 - (1) = -2.$$

$$\therefore \quad y[n] = \begin{cases} 1 & n = 0, \pm 3, \pm 6, \ldots \\ -2 & n = \ldots -5, -2, 1, 4, 7, \ldots \\ 1 & n = \ldots -4, -1, 2, 5, 8, \ldots \end{cases}$$