PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = \tfrac{1}{2}y[n-1] + x[n] \qquad \text{(DIFFERENCE EQUATION)}$$

When the input to the system is

$$x[n] = \begin{cases} +1 & \text{when } n = 0, 1, 2, 3 \\ 0 & \text{when } n < 0 \text{ and } n > 3 \end{cases}$$

determine the functional form for the output signal $y[n]$. Assume that the output signal $y[n]$ is zero for $n < 0$. This is called the *at rest* initial condition for the difference equation.

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SOLUTION

$$y[n] = \tfrac{1}{2}\, y[n-1] + x[n].$$

$$y[0] = \tfrac{1}{2}\, y[-1] + x[0] = 0 + 1 = 1$$

$$y[1] = \tfrac{1}{2}\, y[0] + x[1] = \tfrac{1}{2}(1) + 1 = 3/2$$

$$y[2] = \tfrac{1}{2}\left(\tfrac{3}{2}\right) + x[2] = \tfrac{3}{4} + 1 = 7/4 = 2 - \tfrac{1}{4}$$

$$y[3] = \tfrac{1}{2}\left(7/4\right) + x[3] = \tfrac{7}{8} + 1 = 15/8$$

$$y[4] = \tfrac{1}{2}\left(15/8\right) + x[4] = 15/16 + 0 = 15/16 = 15/2^4$$

$$y[5] = \tfrac{1}{2}\left(15/16\right) + x[5] = 15/32 = 15/2^5$$

$$y[6] = \tfrac{1}{2}\left(15/32\right) + \cancel{x[6]}^{\,0} = 15/64. = 15/2^6$$

$$\Rightarrow \quad y[n] = \begin{cases} 0 & \text{for } n < 0 \\ 2 - \left(\tfrac{1}{2}\right)^n & \text{for } n = 0, 1, 2, 3 \\ 15\left(\tfrac{1}{2}\right)^n & \text{for } n = 4, 5, 6, \ldots \end{cases}$$

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