PROBLEM:

For a second-order filter (reson), determine the gain factor G needed to make the maximum gain of the filter equal to one. In other words, make the maximum magnitude of the frequency response equal to one. Recall that the z-transform of the reson is:

$$\hat{H}(z) = \frac{G}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$$

(a) Assume that $r > 0.8$ and that θ lies between $\pi/6$ and $5\pi/6$. Use approximations to derive a formula for G.

(b) In addition, find the value(s) of ω where the peak occurs.

(c) Then evaluate G when the poles of $\hat{H}(z)$ are $\frac{2}{3} \pm j\frac{2}{3}$.

(d) Explain why your formula for G is no good when the poles are $0.95 \pm j0.05$.

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

(a) PEAK of $H(e^{j\hat{\omega}})$ will be at $\hat{\omega} = \theta$

$$H(e^{j\hat{\omega}}) = \frac{G}{1 - 2r\cos\theta\, e^{-j\hat{\omega}} + r^2 e^{-j2\hat{\omega}}}$$

$$= \frac{G}{(1 - re^{j\theta}e^{-j\hat{\omega}})(1 - re^{-j\theta}e^{-j\hat{\omega}})}$$

$$|H(e^{j\hat{\omega}})| = \frac{|G|}{|1 - re^{j(\theta-\hat{\omega})}| \cdot |1 - re^{-j(\theta+\hat{\omega})}|}$$

- evaluate at $\hat{\omega} = \theta$
- assume $G > 0$

$$|H(e^{j\hat{\omega}})|\Big|_{\omega=\theta} = \frac{G}{|\underbrace{1-r}_{\text{THIS TERM IS SMALL}}| \cdot |\underbrace{1 - re^{-j2\theta}}_{\text{THIS ONE IS LARGE}}|}$$

$$|1 - re^{-j2\theta}| = \sqrt{(1 - r\cos 2\theta)^2 + r^2\sin^2 2\theta}$$

$$= \sqrt{1 + r^2 - 2r\cos 2\theta}$$

- Now set $|H(e^{j\theta})| = 1$ & solve for G.

$$G = (1-r)\sqrt{1 + r^2 - 2r\cos 2\theta}$$

(b) Peaks at $\hat{\omega} = \theta$ AND $\hat{\omega} = -\theta$

Problem CONTINUED

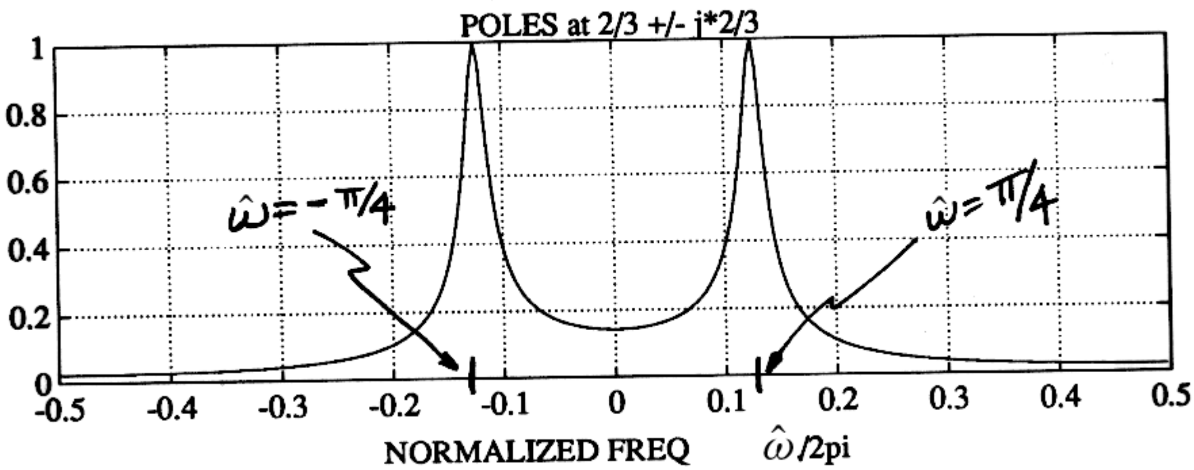
(c) poles at $\frac{2}{3} \pm j\frac{2}{3} = \frac{2\sqrt{2}}{3} e^{\pm j\pi/4}$

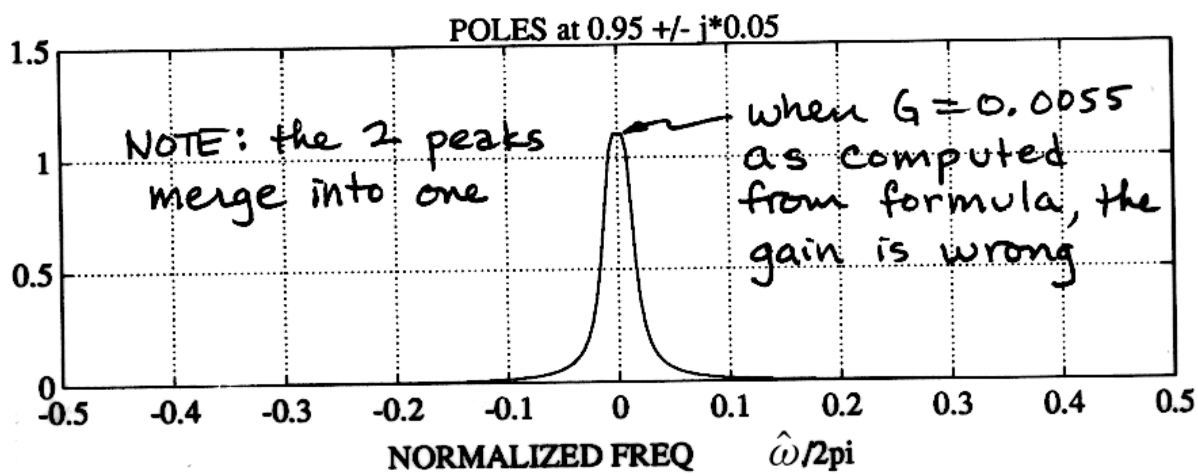
$\therefore \; \theta = \pi/4$ and $r = \frac{2\sqrt{2}}{3} = 0.943$

$G = 0.0786$

see plot below.

(d) The angle of $0.95 \pm j0.05$ is 3° or $\frac{1}{19}$ rad.
so the pos. freq peak interacts with
negative freq. peak





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