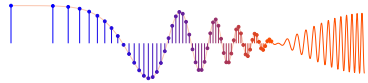


PROBLEM:

For the following second-order filter

$$\hat{H}(z) = \frac{5(1 - z^{-2})}{1 - 1.5\,z^{-1} + 0.81\,z^{-2}}$$

(a) Find the poles and zeros of $\hat{H}(z)$ and plot their location in the z-plane.

(b) Make a sketch of the magnitude of the frequency response of this system. Label the values of $H(\omega)$ at $\omega = 0, \frac{1}{2}\pi, \pi$. Determine the magnitude at the peak.

(c) Write the time-domain description of this system—in the form of a difference equation.

(d) Make a plot of the first 5–10 values in the impulse response of the system (output when $x[n] = \delta[n]$).

(e) Determine the mathematical form for the impulse response to show its behavior versus n.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$H(z) = \frac{5(1 - z^{-2})}{1 - 1.5z^{-1} + 0.81z^{-2}}$$

(a) numerator roots @ $z = +1$ and $z = -1$
these are <u>zeros</u> of $H(z)$

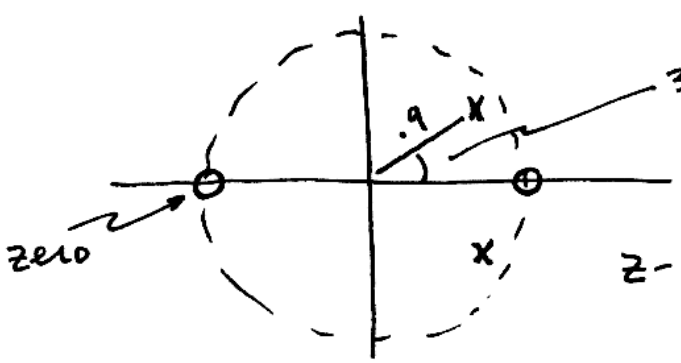
denominator roots at:

$r^2 = 0.81 \Rightarrow r = .9$

$-2r\cos\theta = -1.5$

$\Rightarrow \cos\theta = \frac{.75}{.9} \Rightarrow \theta = \pm 33.6^\circ$ or $\pm 0.186\pi$ rad

$\Rightarrow$ POLES at $0.9e^{\pm j0.186\pi}$



(b) $H(e^{j\hat{\omega}}) = \frac{5(1 - e^{-j2\hat{\omega}})}{1 - 1.5e^{-j\hat{\omega}} + 0.81e^{-j2\hat{\omega}}}$

at $\hat{\omega} = 0$

$H(e^{j\hat{\omega}}) = \frac{5(1-1)}{\text{den}} = 0$

at $\hat{\omega} = \pi$ $H(e^{j\pi}) = \frac{5(1-(-1)^2)}{\text{den}} = 0$

at $\hat{\omega} = \pi/2$

$H(e^{j\hat{\omega}}) = \frac{5(1-(-j)^2)}{1 - 1.5(-j) + 0.81(-j)^2} = \frac{5(2)}{1 + 1.5j - .81}$

$= \frac{10}{.19 + j1.5} = 6.6e^{-j0.46\pi}$

Problem CONTINUED

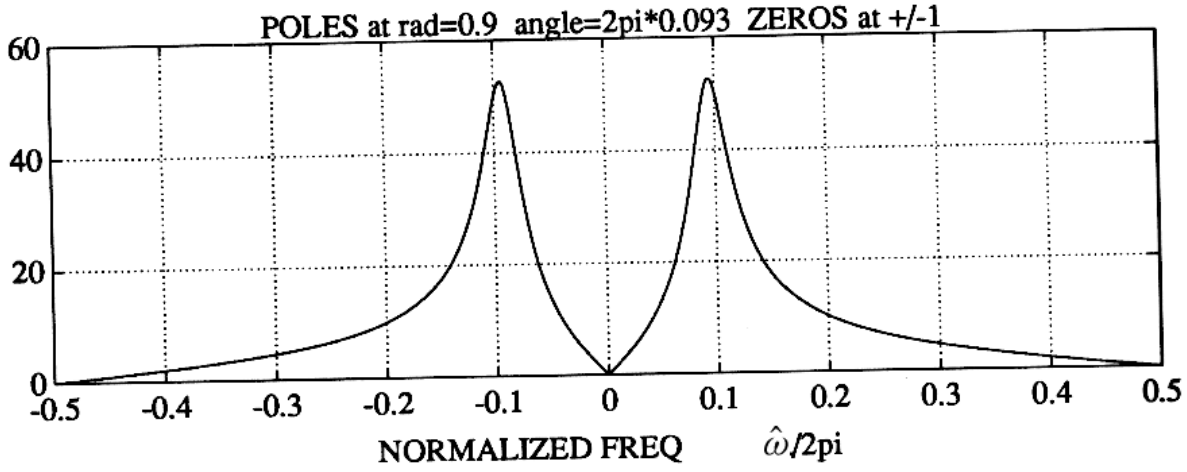
(b) Magnitude at peak $\hat{\omega} = \theta = 0.186\pi$ radians.

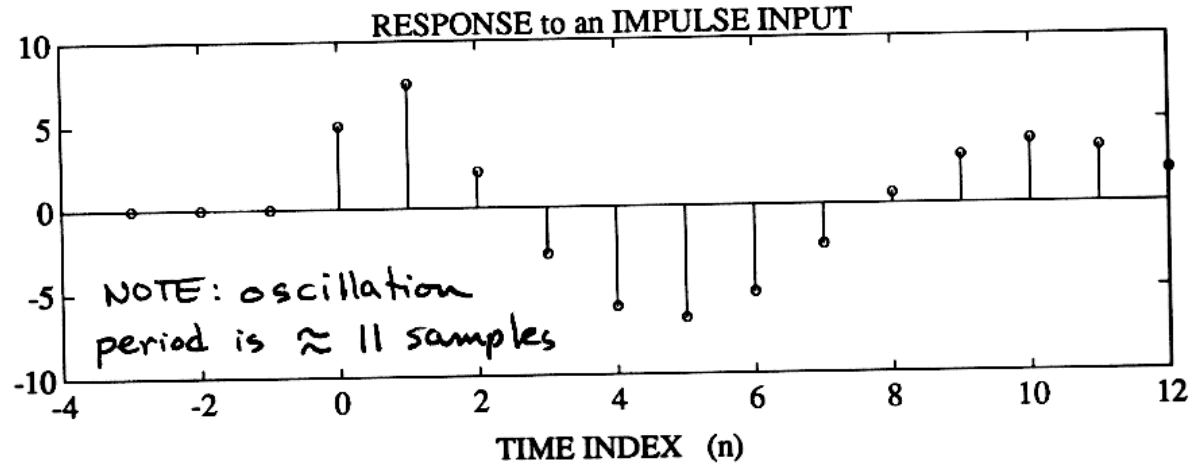
use Problem 7.1 with mod for numerator.

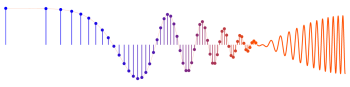
$$\text{peak value} = \frac{5\left|\left(1 - e^{-j\,0.372\pi}\right)\right|}{(1-r)\left(\sqrt{1 + r^2 - 2r\cos 2\theta}\right)}$$

$r = 0.9$

$$\text{peak is.} \quad \frac{5 \cdot (1.1033)}{(0.1)(1.05)} \approx 52.5$$







Problem CONTINUED

(c) $y[n] = 1.5y[n-1] - 0.81y[n-2] + 5x[n] - 5x[n-2]$

$y[0] = 1.5(0) - .81(0) + 5(1) - 5(0) = 5$

$y[1] = 1.5(5) - .81(0) + 5(0) - 5(0) = 7.5$

$y[2] = 1.5(7.5) - .81(5) + 5(0) - 5(1) = 2.2$

- For $n > 2$, the input $x[n] = \delta[n]$ no longer has any affect.

$y[3] = 1.5(2.2) - .81(7.5) = -2.775$

$y[4] = 1.5(-2.775) - .81(2.2) = -5.9445$

After $n > 2$, the output is composed of <u>two</u> signals: $(\text{poles})^n$

$$y[n] = \underbrace{A_1 e^{j\varphi_1}}_{\alpha_1} (.9e^{j0.186\pi})^n + \underbrace{A_2 e^{j\varphi_2}}_{\alpha_2} (.9)^n e^{-j0.186\pi n}$$

α_1, α_2 ← COMPLEX CONSTANTS

Can write 2 EQNS IN 2 UNKNOWNS.

To solve for α_1 & α_2.

$$-2.775 = y[3] = \alpha_1 (.9)^3 e^{j0.558\pi} + \alpha_2 (.9)^3 e^{-j0.558\pi}$$

$$-5.9445 = y[4] = \alpha_1 (.9)^4 e^{j0.744\pi} + \alpha_2 (.9)^4 e^{-j0.744\pi}$$

SOLUTION for α_1 & α_2 is:

$\alpha_1 = 5.586 + j0.884 = 5.656 e^{j0.05\pi}$

$\alpha_2 = 5.586 - j0.884 = 5.656 e^{-j0.05\pi}$

NOTE: $\alpha_2 = \alpha_1^*$

Problem

FINAL FORMULA

$$y[n] = 5.656\, e^{j0.05\pi} e^{j0.186\pi n} \cdot .9^n + 5.656\, e^{-j0.05\pi} \cdot .9^n e^{-j.186\pi n}$$

$$= 5.656\,(.9)^n \left(e^{j(0.186\pi n + 0.05\pi)} + e^{-j(0.186\pi n + 0.05\pi)} \right)$$

$$y[n] = 5.656\,(.9)^n \left(2\cos(0.186\pi n + 0.05\pi)\right)$$

$$= 11.312\,(0.9)^n \cos(0.186\pi n + 0.05\pi)$$

formula is good for $n > 2$.