PROBLEM:

A second-order filter has poles at $z = \frac{1}{2} \pm j\frac{1}{2}\sqrt{3}$ and two zeros at $z = \pm 1$.

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) Determine the response of the system to an input that is an impulse. Give the answer as a formula and as a plot.

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SOLUTION

(a) $H(z) = \dfrac{(1-z^{-1})(1+z^{-1})}{\left(1-(\frac{1}{2}+j\frac{1}{2}\sqrt{3})z^{-1}\right)\left(1-(\frac{1}{2}-j\frac{1}{2}\sqrt{3})z^{-1}\right)}$

$= \dfrac{1-z^{-2}}{1-z^{-1}+z^{-2}}$ poles @ $1e^{j\pi/3}$ & $1e^{-j\pi/3}$

$\therefore$ $y[n] = y[n-1] - y[n-2] + x[n] - x[n-2]$.

(b) $y[0] = 0 - 0 + 1 - 0 = 1$

$y[1] = 1 - 0 + 0 - 0 = 1$ REPEATS.

$y[2] = 1 - 1 + 0 - 1 = -1$

After $n>2$, the input is gone

$y[3] = -1 - 1 = -2$

$y[4] = -2-(-1) = -1$

$y[5] = -1-(-2) = 1$

$y[6] = 1-(-1) = 2$

$y[7] = 2-1 = 1$

$y[8] = 1-2 = -1$

For $n>2$: $y[n] = \alpha_1 e^{j\pi n/3} + \alpha_2 e^{-j\pi n/3}$

Make 2 eqns in 2 unknowns.

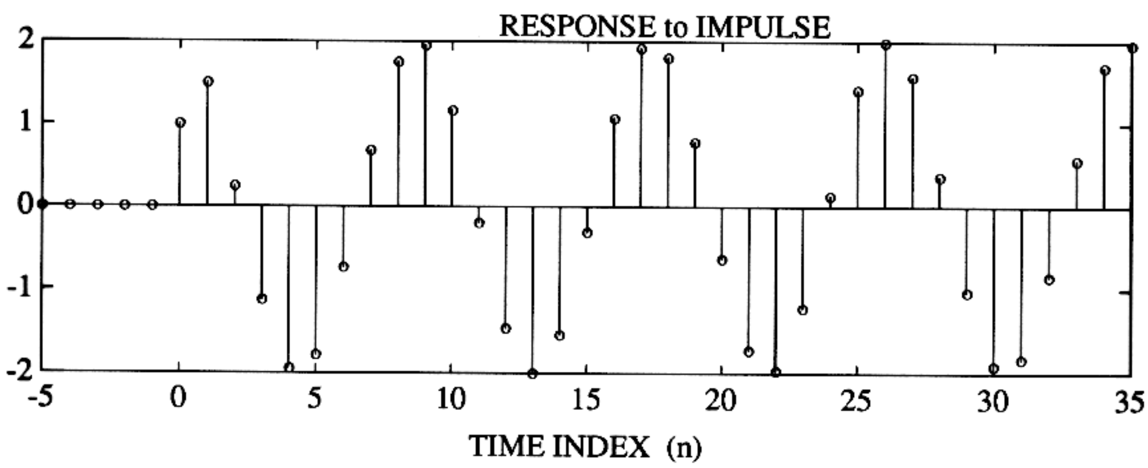
$-2 = y[3] = \alpha_1 e^{j\pi} + \alpha_2 e^{-j\pi} = -\alpha_1 - \alpha_2$

$-1 = y[4] = \alpha_1 e^{j4\pi/3} + \alpha_2 e^{-j4\pi/3}$

Solution is $\alpha_1 = 1$ & $\alpha_2 = 1$

$\therefore$ $y[n] = e^{j\pi n/3} + e^{-j\pi n/3} = 2\cos\frac{\pi n}{3}$

This formula applies for $n>0$.



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