PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = -0.9\, y[n-5] + x[n] \qquad \text{(DIFFERENCE EQUATION)} \qquad (1)$$

(a) When the input to the system is the impulse signal:

$$x[n] = \begin{cases} +1 & \text{when } n = 0 \\ 0 & \text{when } n \neq 0 \end{cases}$$

make a plot of the output signal $y[n]$ to show its important characteristics. Assume that the output signal is zero for $n < 0$.

(b) Find the z-transform operator representation for the system in (1)

(c) Find the poles of the system and plot in the z-plane.

(d) Derive a formula for the frequency response of the system.

(e) Sketch the magnitude of the frequency response.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$y[n] = -0.9\,y[n-5] + x[n].$$

(a) $y[0] = -0.9(0) + 1 = 1$

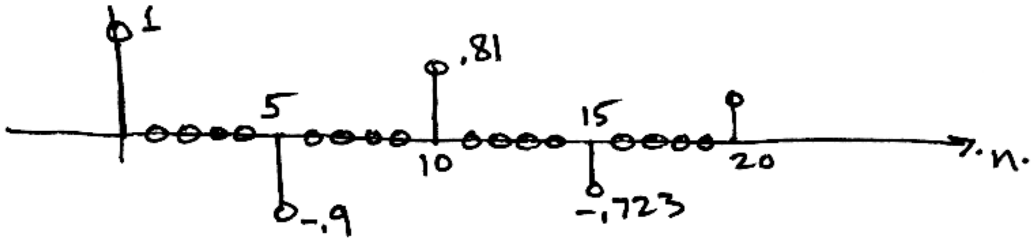
$y[1] = y[2] = 0$

$y[3] = y[4] = 0$

$y[5] = -0.9(1) + 0 = -0.9$

$y[6] = y[7] = y[8] = y[9] = 0$

$y[10] = 0.81$



(b) $$H(z) = \frac{1}{1 + 0.9z^{-5}}$$

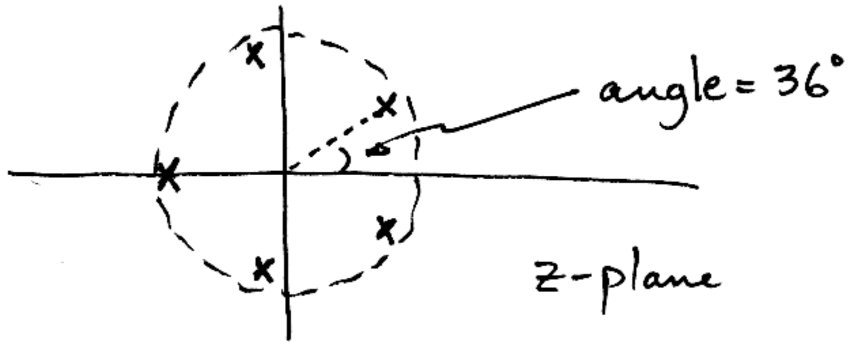
(c) Poles at roots of $1 + 0.9z^{-5} = 0$

$\Rightarrow z^5 + 0.9 = 0$

$\Rightarrow z^5 = -0.9 = 0.9\,e^{j(\pi + 2\ell\pi)}$

$\therefore \quad z = (0.9)^{1/5}\, e^{j(\pi/5 + 2\ell\pi/5)}$ — angles are $\pi/5, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

$(0.9)^{1/5} = .98$



PROBLEM CONT

(d) $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + 0.9\,e^{-j5\hat{\omega}}}$

AT $\omega = 0$

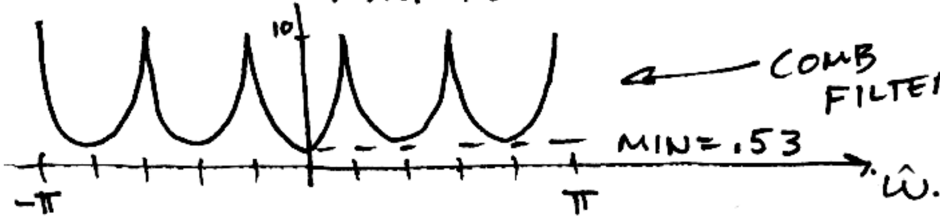
$H(e^{j0}) = \frac{1}{1.9} = .53$

when $\hat{\omega} = \frac{\pi}{5}$ $H(e^{j\pi/5}) = \dfrac{1}{1+.9e^{-j\pi}} = \dfrac{1}{1-.9} = 10$

PEAK

other peaks at $\hat{\omega} = 3\pi/5$, π, $\frac{7\pi}{5}$, & $9\pi/5$.

MAG. PLOT.



FREQ RESPONSE of COMB FILTER