PROBLEM:

Given a feed-forward filter defined via the weighted average:

$$y[n] = x[n] + \sqrt{2}x[n-1] + x[n-2]$$

(a) Find the z-transform operator description for this system.

(b) Calculate the roots of the z-transform polynomial. Express your answer in polar form.

(c) Explain in words the significance of the root locations when processing an input signal of the form $x[n] = e^{j\omega n}$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Given a feed-forward filter defined via the weighted average:

$$y[n] = x[n] + \sqrt{2}x[n-1] + x[n-2]$$

(a) Find the z-transform operator description for this system.

$$Y(z) = X(z) + \sqrt{2}\, z^{-1} X(z) + z^{-2} X(z)$$

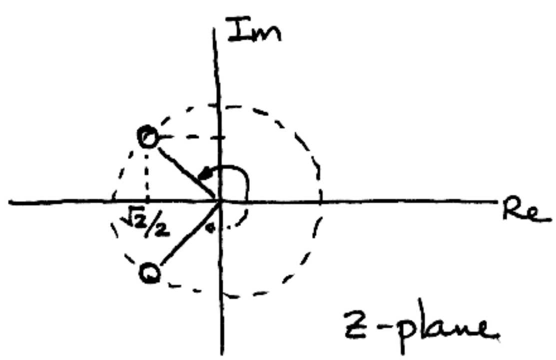
$$= \left(1 + \sqrt{2}\, z^{-1} + z^{-2}\right) X(z)$$

$$\therefore\ H(z) = 1 + \sqrt{2}\, z^{-1} + z^{-2}$$

(b) Calculate the roots of the z-transform polynomial. Express your answer in polar form.

$$\text{ROOTS} = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2} = \frac{-\sqrt{2} \pm j\sqrt{2}}{2}$$

$$= 1\, e^{\pm j 3\pi/4}$$

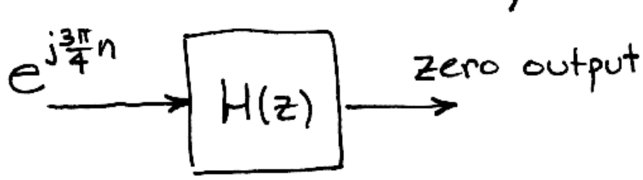


(c) Explain in words the significance of the root locations when processing an input signal of the form $x[n] = e^{j\hat{\omega} n}$

$$H(z) = 0 \quad \text{at} \quad z = 1\, e^{\pm j 3\pi/4}$$

$\Rightarrow$ when $\hat{\omega} = \pm 3\pi/4$, the output will be ZERO

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega} = \frac{3\pi}{4}} = 0$$



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