PROBLEM:

A linear time-invariant system has system function

$$H(z) = (1 + z^{-1})(1 - z^{-1}) = 1 - z^{-2}$$

The input to this system is

$$x[n] = 5 - 4\delta[n] + 10\cos(0.5\pi n + \pi/4)$$

Determine the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Use superposition

$$x[n] = 5 - 4\delta[n] + 10\cos(0.5\pi n + \pi/3).$$

$$\underline{\text{IN}} \qquad\qquad\qquad \underline{\text{OUT}}$$

$$5 \longrightarrow \mathcal{H}(0)\cdot 5 = H(1)\cdot 5 = 0$$

$$-4\delta[n] \longrightarrow -4h[n] = -4\delta[n] + 4\delta[n-2]$$

$$10\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{3}\right) \longrightarrow 5\mathcal{H}\left(\tfrac{\pi}{2}\right)e^{j\pi/3}e^{j\frac{\pi}{2}n} + 5\mathcal{H}\left(-\tfrac{\pi}{2}\right)e^{-j\frac{\pi}{3}}e^{-j\frac{\pi}{2}n}$$

$$H(e^{j\hat{\omega}}) = 1 - z^{-2}\Big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j2\hat{\omega}}$$

$$H(e^{j0}) = 1 - e^{-j0} = 0$$

$$\mathcal{H}(\pi/2) = 1 - e^{-j\pi} = 2$$

$$\mathcal{H}(-\pi/2) = 1 - e^{+j\pi} = 2$$

$\underline{\underline{\text{NO}}}$ phase change

So,

$$10\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{3}\right) \longrightarrow 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{3}\right)$$

$y[n]$ is the sum of all these:

$$y[n] = -4\delta[n] + 4\delta[n-2] + 20\cos\left(\tfrac{\pi}{2}n + \pi/3\right)$$