PROBLEM:

A linear time-invariant system has system function given by

$$H(z) = \sum_{n=0}^{5} z^{-n} = \frac{1 - z^{-6}}{1 - z^{-1}}$$

(a) Plot the poles and zeros of $H(z)$ in the complex z-plane.

(b) Use the summation form for $H(z)$ to determine a difference equation that relates the output $y[n]$ to the input $x[n]$ of the above system. The equation should involve only samples of the input.

(c) Use the second form to determine another difference equation that relates the output $y[n]$ to the input $x[n]$ of the above system. In this case the equation should involve both input and output samples.

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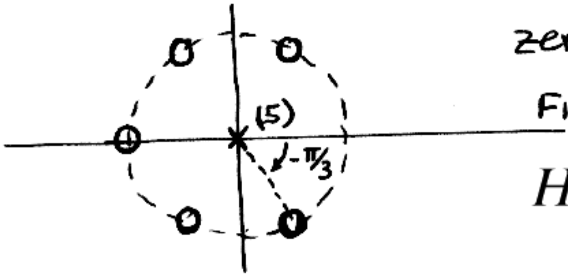
SOLUTION

$$H(z) = \frac{1 - z^{-6}}{1 - z^{-1}}$$

NOTE: at $z = 1$
$H(z) = 6$

(a) Zeros are roots of $1 - z^{-6} = 0$ or $z^6 = 1$



zeros at $e^{j\frac{\pi}{3}\ell}$, $\ell = 1, 2, 3, 4, 5$

Five poles at $z = 0$

$$H(z) = \frac{z^6 - 1}{z^5(z-1)}$$

(b) $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$

$\Rightarrow$ difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5]$$

(c) $H(z) = \dfrac{1 - z^{-6}}{1 - z^{-1}}$

$$y[n] = y[n-1] + x[n] - x[n-6]$$

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