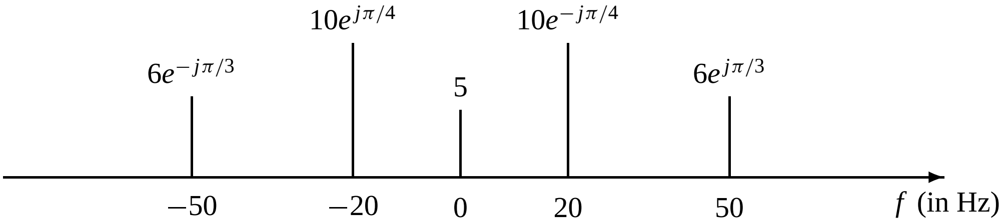
PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.

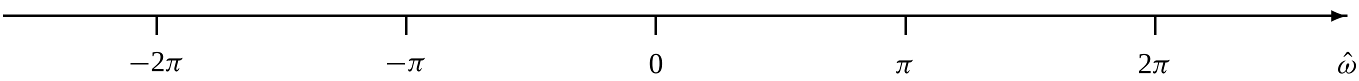


(a) Which side ($f > 0$ or $f < 0$) of the spectrum plot corresponds to complex phasors rotating in the clockwise direction?

(b) Write an equation for $x(t)$ as a sum of complex rotating phasors. (These phasors may rotate clockwise, counter-clockwise or not at all.)

(c) Write an equation for $x(t)$ as a sum of real functions.

(d) Suppose that the signal is sampled to produce the sequence $x[n] = x(nT_s)$, where $f_s = 1/T_s = 200$. Below, sketch the spectrum of the sampled signal (i.e., show the alias frequencies) as a function of both cyclic frequency $-f_s < f < f_s$ and normalized frequency $\hat{\omega} = 2\pi f T_s$ for normalized frequencies $-2\pi < \hat{\omega} < 2\pi$. *Label the axis carefully. You* **do not** *have to write out the complex amplitude, just plot the spectral line at its correct location.*



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

— Need to know how to deal with the spectrum

(a) clockwise rotation ⇒ negative frequency $(f < 0)$

(b) Going left to right in the spectrum,

$$x(t) = 6e^{-j\pi/3}e^{-j100\pi t} + 10e^{j\pi/4}e^{-j40\pi t} + 5 + 10e^{-j\pi/4}e^{j40\pi t} + 6e^{j\pi/3}e^{j100\pi t}$$

(c) Combining complex conjugate terms gives

$$x(t) = 5 + 20\cos(40\pi t - \pi/4) + 12\cos(100\pi t + \pi/3)$$

(d) When we sample with $f_s = 200$ the alias frequencies are at $0 + k200$, $\pm 20 + k200$ & $\pm 50 + k200$.

The "normalized" frequencies are $\hat{\omega} = 2\pi f/200$

or $0 + k$, $.2\pi + k2\pi$ & $.5\pi + k2\pi$.