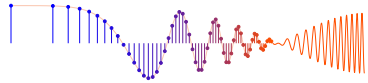


PROBLEM:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - x[n-4]$$

(a) Find its system function $H(z)$.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) Find the frequency response $H(e^{j\hat{\omega}})$ and express it in polar form (magnitude and phase).
Remember the "trick" $(1 - e^{-j\theta}) = e^{-j\theta/2}(e^{j\theta/2} - e^{-j\theta/2})$.

(d) Sketch $|H(e^{j\hat{\omega}})|$ for $-\pi < \hat{\omega} < \pi$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

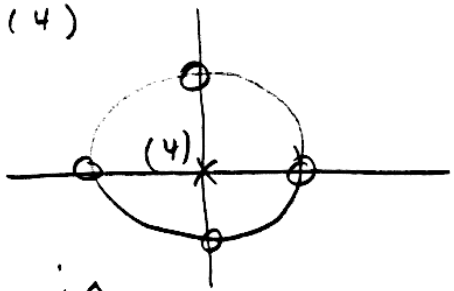
SOLUTION

$y[n] = x[n] - x[n-4]$

(a) By inspection $H(z) = 1 - z^{-4} = (z^4 - 1)/z^4$

(b) zeros: $z^4 = 1 \Rightarrow e^{j\frac{2\pi}{4}k}$ $k = 0, 1, 2, 3$

poles: $z^4 \Rightarrow z = 0$ (4)



(c) $H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j\hat{\omega}4}$

$= e^{-j\hat{\omega}2}(e^{j\hat{\omega}2} - e^{-j\hat{\omega}2}) = 2j e^{-j\hat{\omega}2}\sin(\hat{\omega}2)$

$|H(e^{j\hat{\omega}})| = 2|\sin(2\hat{\omega})| \qquad \angle H(e^{j\hat{\omega}}) = \frac{\pi}{2} - 2\hat{\omega}$

(d)