PROBLEM:

A linear time-invariant system has system function

$$H(z) = \sum_{k=0}^{3} (0.5)^k z^{-k}$$

(a) Write the difference equation relating the output $y[n]$ to the input $x[n]$.

(b) Determine the response of this system to a unit impulse input; i.e., find the output $y[n] = h[n]$ when the input is $x[n] = \delta[n]$. Plot $h[n]$ as a function of n.

(c) Find the poles and zeros of $H(z)$ and plot them in the complex z-plane.

Remember the formula $\displaystyle\sum_{k=0}^{L-1} \alpha^k = \frac{1-\alpha^L}{1-\alpha}$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

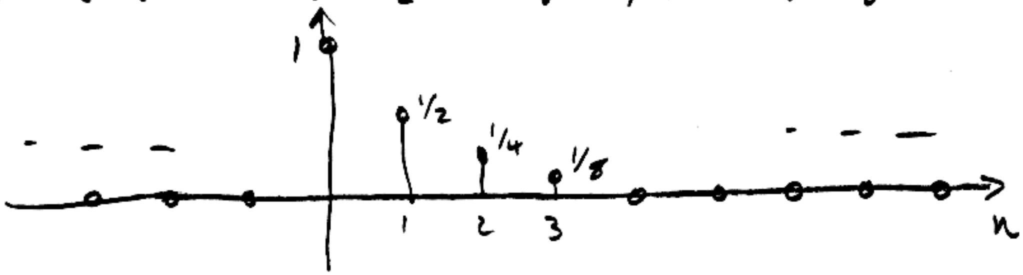
SOLUTION

$$H(z) = \sum_{k=0}^{3} (.5)^k z^{-k}$$

(a) By inspection $y[n] = \sum_{k=0}^{3} (.5)^k x[n-k]$

$$y[n] = x[n] + \tfrac{1}{2} x[n-1] + \tfrac{1}{4} x[n-2] + \tfrac{1}{8} x[n-3]$$

(b) $h[n] = \delta[n] + \tfrac{1}{2}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{8}\delta[n-3]$

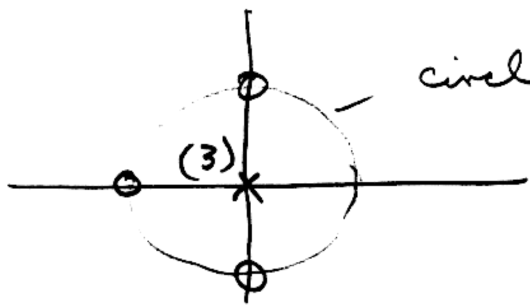


(c) $H(z) = \dfrac{1-(.5z^{-1})^4}{1-.5z^{-1}} = \left(\dfrac{z^4-(.5)^4}{z-.5}\right) z^{-3}$

Zeros: $z^4 = (.5)^4 \Rightarrow .5e^{j\frac{\pi}{4}k}$ $\quad k = 0, 1, 2, 3$

Poles $z^{-3} \Rightarrow 3$ at $z = 0$ & $z = .5$

circle radius .5



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