PROBLEM:

Suppose that a system is defined by the following operator

$$\hat{H}(z) = (1 - e^{j\pi/4} z^{-1})(1 - e^{-j\pi/4} z^{-1})(1 + z^{-2})$$

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) If the input to this system is an impulse $x[n] = \delta[n]$, determine the impulse response $h[n]$ for $n \geq 0$.

(c) Derive simple formulas for the magnitude of the frequency response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$, and the phase response versus $\hat{\omega}$. These formulas must contain no complex terms and no square roots.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Suppose that a system is defined by the following operator

$$H(z) = (1 - e^{j\pi/4}z^{-1})(1 - e^{-j\pi/4}z^{-1})(1 + z^{-2})$$

(a) Write the time-domain description of this system—in the form of a difference equation.

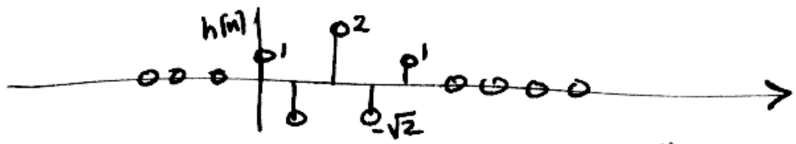
$$H(z) = (1 - \underbrace{2\cos\pi/4}_{\sqrt{2}}\, z^{-1} + z^{-2})(1 + z^{-2})$$

$$= 1 - \sqrt{2}\, z^{-1} + 2z^{-2} - \sqrt{2}\, z^{-3} + z^{-4}$$

$$\Rightarrow \quad y[n] = x[n] - \sqrt{2}\, x[n-1] + 2\, x[n-2] - \sqrt{2}\, x[n-3] + x[n-4]$$

(b) If the input to this system is an impulse $x[n] = \delta[n]$, determine the impulse response $h[n]$ for $n \geq 0$.

$$h[n] = \delta[n] - \sqrt{2}\,\delta[n-1] + 2\,\delta[n-2] - \sqrt{2}\,\delta[n-3] + \delta[n-4].$$



(c) Derive simple formulas for the magnitude of the frequency response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$, and the phase response versus $\hat{\omega}$. These formulas must contain no complex terms and no square roots.

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

$$= 1 - \sqrt{2}\, e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - \sqrt{2}\, e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}} - \sqrt{2}\, e^{j\hat{\omega}} + 2 - \sqrt{2}\, e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right).$$

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}\left(2\cos 2\hat{\omega} - 2\sqrt{2}\cos\hat{\omega} + 4\right).$$

$$\therefore \quad \varphi(\hat{\omega}) = -2\hat{\omega} \longleftarrow \boxed{\text{LINEAR PHASE}}$$

$$|H(e^{j\hat{\omega}})| = 4 - 2\sqrt{2}\cos\hat{\omega} + 2\cos 2\hat{\omega}$$