PROBLEM:

The intention of the following MATLAB program is to synthesize a sinusoid that could be played out through a D/A converter. The synthesis is done by using a recursive (feedback) filter, implemented via MATLAB's `filter` function.

```
imp = [ 1, zeros(1,9999)];
bb = [0   1   0];
aa = [1  -1.9  1];
xn = filter( bb, aa, imp );
```

(a) Determine the poles of the synthesis filter.

(b) Determine a formula for $x[n]$, the signal contained in the vector `xn`. This formula should give numerical values for the amplitude, phase and frequency of $x[n]$.

(c) If this signal is played out through a D-A converter with $f_s = 8$ kHz, what frequency (in Hertz) will be heard?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

The intention of the following MATLAB program is to synthesize a sinusoid that could be played out through a D/A converter. The synthesis is done by using a recursive (feedback) filter, implemented via MATLAB's `filter` function.

```
imp = [ 1, zeros(1,9999)];
bb = [0    1    0];
aa = [1   -1.9   1];
xn = filter( bb, aa, imp );
```

$$H(z) = \frac{z^{-1}}{1 - 1.9z^{-1} + z^{-2}}$$

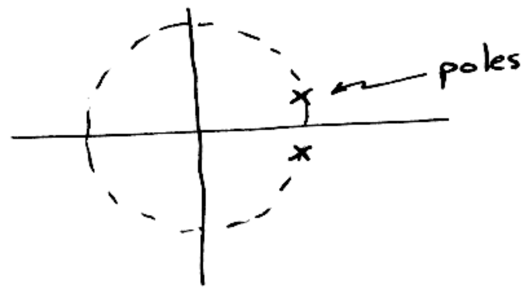
(a) Determine the poles of the synthesis filter.

$$1 - 1.9z^{-1} + z^{-2} = A(z)$$

$$\text{poles} = \frac{1.9 \pm \sqrt{3.61 - 4}}{2}$$

$$\cong .95 \pm j\,0.3122$$

$$= 1\,e^{\pm j\,0.101\pi}$$



(b) Determine a formula for $x[n]$, the signal contained in the vector `xn`. This formula should give numerical values for the amplitude, phase and frequency of $x[n]$.

$$x[n] = 1.9x[n-1] - x[n-2] + \delta[n-1].$$

n	<0	0	1	2	3	4	5	6	7	8
$x[n]$	0	0	1	1.9	2.61	3.059	3.202	3.025	2.545	1.811

peak

$$x[n] = A\cos(\hat{\omega}_0 n + \varphi)$$

PERIOD

$$\omega_0 = 0.101\pi = 2\pi(0.0505) \cong 2\pi/19.8$$

$$\varphi = -\pi/2 \quad (\text{because } x[0]=0) \Rightarrow x[n] = A\sin(\hat{\omega}_0 n)$$

$$\Rightarrow 1 = A\sin\hat{\omega}_0$$

$$A = 3.2026 \qquad \Rightarrow A = 1/\sin\hat{\omega}_0 = 3.2026$$

(c) If this signal is played out through a D-A converter with $f_s = 8$ kHz, what frequency (in Hertz) will be heard?

$$\text{FREQ} = \frac{\hat{\omega}_0}{2\pi} \times 8000 = (0.0505)\,8000 = 404.3 \text{ Hz}$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.