

PROBLEM:

Suppose that a system is defined by the following operator

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1}}$$

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) Derive a simple formula for the "magnitude squared" of the frequency response $|H(e^{j\hat{\omega}})|^2$ versus $\hat{\omega}$. This formula must contain no complex terms and no square roots.

(c) Is this system a lowpass or highpass filter? EXPLAIN

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$$y[n] = \frac{1}{2}y[n-1] + x[n] + 2x[n-1] + x[n-2]$$

(b) Derive a simple formula for the "magnitude squared" of the frequency response $|H(e^{j\hat{\omega}})|^2$ versus $\hat{\omega}$. This formula must contain no complex terms and no square roots.

$$|H(e^{j\hat{\omega}})|^{2} = \frac{1+2e^{j\hat{\omega}}+e^{j^{2}\hat{\omega}}}{1-\frac{1}{2}e^{-j\hat{\omega}}} \cdot \frac{1+2e^{+j\hat{\omega}}+e^{+j^{2}\hat{\omega}}}{1-\frac{1}{2}e^{+j\hat{\omega}}}$$

$$= \frac{e^{-j\hat{\omega}}\left(e^{+j\hat{\omega}}+2+e^{-j\hat{\omega}}\right)e^{j\hat{\omega}}\left(e^{-j\hat{\omega}}+2+e^{+j\hat{\omega}}\right)}{1+\frac{1}{4}-\frac{1}{2}e^{-j\hat{\omega}}-\frac{1}{2}e^{+j\hat{\omega}}}$$

$$= \frac{(2+2\cos\hat{\omega})^{2}}{54-\cos\hat{\omega}}$$

$$\frac{OR}{10(-j\hat{\omega})^{12}} = 6+8\cos\hat{\omega}+2\cos 2\hat{\omega}$$

$$|H(e^{j\hat{\omega}})|^2 = \frac{6 + 8\cos\omega + 2\cos 2\omega}{5/4 - \cos\hat{\omega}}$$

(c) Is this system a lowpass or highpass filter? EXPLAIN

POLE (a)
$$Z = +\frac{1}{2}$$

2 ZEROS (a) $Z = -1$
 $H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = H(z)\Big|_{Z=1} = \frac{4}{\frac{1}{2}} = 8$
 $H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi} = H(z)\Big|_{Z=-1} = 0$

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