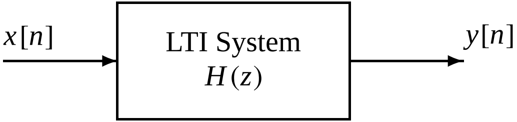
PROBLEM:

Consider the following linear time-invariant system:



The system function of the linear time-invariant filter is given by the formula

$$H(z) = (1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})(1 - 0.9e^{j\pi/3}z^{-1})(1 - 0.9e^{-j\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

(b) Plot the poles and zeros of $H(z)$ in the complex z-plane.

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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SOLUTION

(a) $H(z) = (1-z^{-1})\underbrace{(1-e^{j\pi/2}z^{-1})(1-e^{-j\pi/2}z^{-1})}_{1-2\cos\frac{\pi}{2}z^{-1}+z^{-2} \;=\; 1+z^{-2}}\underbrace{(1-0.9e^{j\pi/3}z^{-1})(1-0.9e^{-j\pi/3}z^{-1})}_{1-1.8\cos\frac{\pi}{3}z^{-1}+z^{-2}(.81) \;=\; 1-.9z^{-1}+(0.81)z^{-2}}$

$$H(z) = \underbrace{(1-z^{-1})(1+z^{-2})}_{(1-z^{-1}+z^{-2}-z^{-3})}(1-.9z^{-1}+.81z^{-2}).$$

$$H(z) = 1-1.9z^{-1}+2.71z^{-2}-2.71z^{-3}+1.71z^{-4}-.81z^{-5}$$

$\therefore$ $y[n] = x[n] - 1.9x[n-1] + 2.71x[n-2] - 2.71x[n-3] + 1.71x[n-4]$
$- 0.81x[n-5]$

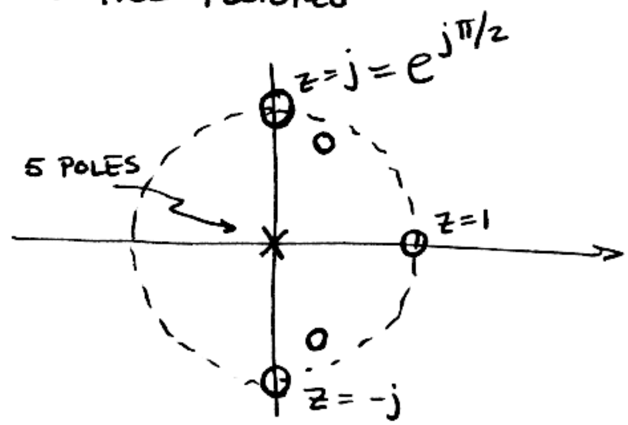
(b) Zeros can be found from the factored form:

$z = 1,\ j,\ -j,\ 0.9e^{j\pi/3}$

$0.9e^{-j\pi/3}$ are the ZEROS

POLES @ $z=0$

5 of them



(c) The zeros <u>on</u> the UNIT CIRCLE mean that $H(e^{j\hat{\omega}}) = 0$ for some specific freqs (and thus the output for $x[n] = e^{j\hat{\omega}n}$ will be zero).

These freqs are $\hat{\omega} = 0,\ +\pi/2,\ -\pi/2$ which are the angles of the zeros on the UNIT CIRCLE.

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