PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = y[n-1] - y[n-2] + x[n] \qquad \text{(DIFFERENCE EQUATION)} \tag{1}$$

(a) When the input to the system is the impulse signal:

$$x[n] = \begin{cases} +1 & \text{when } n = 0 \\ 0 & \text{when } n \neq 0 \end{cases}$$

determine the output signal $y[n]$. Assume the "at rest" condition: i.e., the output signal is zero for $n < 0$. Since this is the impulse response, use the notation $h[n]$ for this output. It should be easy to generate a few values of $h[n]$ and then see that $h[n]$ is actually periodic for $n \geq 0$.

(b) Determine the frequency $\hat{\omega}_\circ$ of the signal $h[n]$ in part (a). In addition, write a formula for $h[n]$ in the form $A\cos(\hat{\omega}_\circ n + \phi)$ that is valid for $n \geq 0$.

(c) The z-transform operator representation for the system in (1) is

$$H(z) = \frac{1}{1 - z^{-1} + z^{-2}}$$

Find the roots of the denominator polynomial $A(z) = 1 - z^{-1} + z^{-2}$ and relate the angle of the root positions in the z-plane to the frequency of $h[n]$.

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SOLUTION

(a) $y[n] = y[n-1] - y[n-2] + x[n]$

n	<0	0	1	2	3	4	5	6	7	8	9	10	11	12
$x[n]$	0	1	0	0	0	0	0	0	0	0	0	0	0	0
$y[n]$	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1

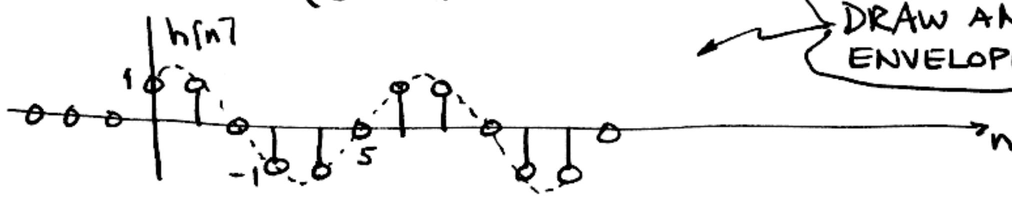
$y[3] = y[2] - y[1]$.

The "period" of $y[n]$ for $n > 0$ is $\boxed{\text{SIX}}$

(b) $\hat{\omega}_0 = 2\pi/\text{period} = 2\pi/6 = \boxed{\pi/3}$

NOTE: $h[n]$ = "IMPULSE RESPONSE"

$h[n] = A\cos\left(\frac{\pi}{3}n + \varphi\right)$



DRAW AN ENVELOPE

when $n = 2$, the argument of the cosine should be $\pi/2$

$$\Rightarrow \; 2\pi/3 + \varphi = \pi/2 \;\Rightarrow\; \varphi = \frac{3-4}{6}\pi = \boxed{-\pi/6.}$$

$$h[0] = A\cos\left(\frac{\pi}{3}n - \pi/6\right)\Big|_{n=0} = A\cos(-\pi/6) = A\frac{\sqrt{3}}{2} = 1$$

$$\Rightarrow \boxed{A = 2/\sqrt{3}} \qquad \boxed{h[n] = \frac{2}{\sqrt{3}}\cos\left(\frac{\pi n}{3} - \pi/6\right)}$$

(c) Roots of $A(z)$

$$1 - z^{-1} + z^{-2} = 0 \;\Rightarrow\; z = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

POLES @ $\begin{cases} \frac{1}{2} + j\frac{\sqrt{3}}{2} = 1e^{j\pi/3} \\ \frac{1}{2} - j\frac{\sqrt{3}}{2} = 1e^{-j\pi/3} \end{cases}$

NOTE: ANGLE OF ROOTS $= \pi/3$ WHICH IS SAME AS $\hat{\omega}_0 = \pi/3$

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