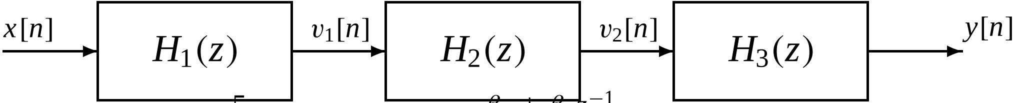
PROBLEM:

In the following cascade of systems, all systems are defined by their transfer functions.



$$H_1(z) = \frac{5}{1 - \frac{1}{2}z^{-1}} \qquad H_2(z) = \frac{\beta_0 + \beta_1 z^{-1}}{1 + \alpha z^{-1}} \qquad H_3(z) = 4z^{-1} - z^{-3}$$

(a) Determine the unknown coefficients $\{\ \beta_0,\ \beta_1,\ \alpha\ \}$ so that the impulse response of the overall system will be $h[n] = 2\delta[n-1] + 3\delta[n-2]$.

(b) Using part (a), determine the overall difference equation that relates $x[n]$ to $y[n]$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

(a) $H(z) = H_1(z)\,H_2(z)\,H_3(z)$

$$= \left(\frac{5}{1-\frac{1}{2}z^{-1}}\right)\left(\frac{\beta_0+\beta_1 z^{-1}}{1+\alpha z^{-1}}\right)\left(4z^{-1}-z^{-3}\right)$$

Want $H(z)$ to be $2z^{-1}+3z^{-2}$

$\Rightarrow$ we need pole-zero cancellations.

Note: $4z^{-1}-z^{-3} = 4z^{-1}\left(1-\frac{1}{4}z^{-2}\right) = 4z^{-1}\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)$

$$\Rightarrow H(z) = \frac{(5)(4z^{-1})\left(1+\frac{1}{2}z^{-1}\right)\left(\beta_0+\beta_1 z^{-1}\right)}{1+\alpha z^{-1}} \stackrel{?}{=} 2z^{-1}+3z^{-2}$$

Cross-multiply and compare terms

$$20\beta_0 z^{-1} + 20\left(\tfrac{1}{2}\beta_0+\beta_1\right)z^{-2} + 10\beta_1 z^{-3} = 2z^{-1} + (2\alpha+3)z^{-2} + 3\alpha z^{-3}$$

$\Rightarrow 20\beta_0 = 2 \Rightarrow \boxed{\beta_0 = 1/10}$

$$\left.\begin{array}{l} 20\beta_1 + 10\beta_0 = 2\alpha+3 \\ 10\beta_1 = 3\alpha \end{array}\right\} \quad \begin{array}{l} 2(3\alpha) + 10\left(\frac{1}{10}\right) = 2\alpha+3 \\ 4\alpha = 2 \Rightarrow \boxed{\alpha = \frac{1}{2}} \end{array}$$

$\Rightarrow \boxed{\beta_1 = \frac{3}{10}\alpha = \frac{3}{20}}$

(b) Since $H(z) = 2z^{-1}+3z^{-2}$, the filter is FIR.

$$y[n] = 2x[n-1] + 3x[n-2]$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.