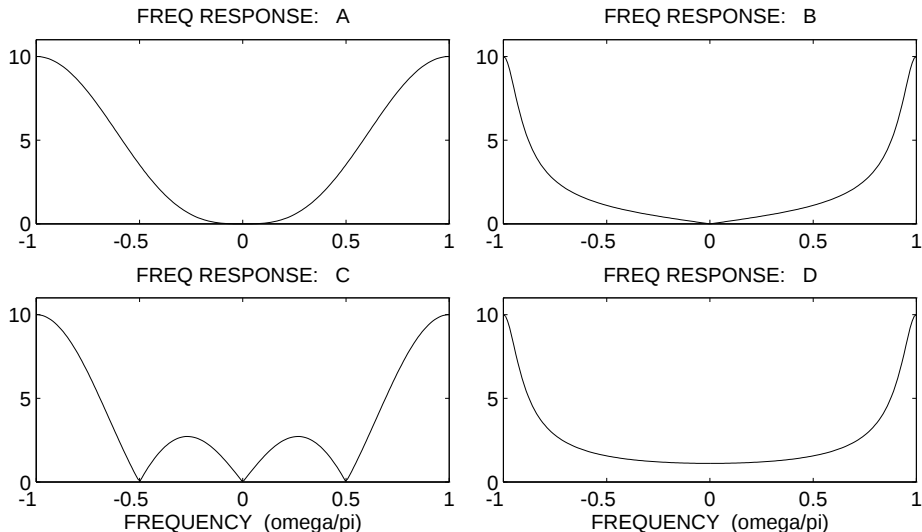




PROBLEM:



For the following systems, determine which frequency response plot (A, B, C, D) matches.

- (a) For the following system derive a simple expression for $|H(e^{j\hat{\omega}})|$, and select the appropriate plot.

$$y[n] = 2\frac{1}{2}x[n] - 2\frac{1}{2}x[n-1] + 2\frac{1}{2}x[n-2] - 2\frac{1}{2}x[n-3]$$

- (b) For the following system

$$y[n] = -0.8y[n-1] + x[n] - x[n-1]$$

derive a simple real-valued expression for $|H(e^{j\hat{\omega}})|$, and select the appropriate plot.



$$(a) \quad y[n] = \frac{5}{2} (x[n] - x[n-1] + x[n-2] - x[n-3])$$

$$H(e^{j\hat{\omega}}) = \frac{5}{2} (1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}})$$

$$= \frac{5}{2} e^{-j3\hat{\omega}/2} \left(e^{+j3\hat{\omega}/2} - e^{j\hat{\omega}/2} + e^{-j\hat{\omega}/2} - e^{-j3\hat{\omega}/2} \right)$$

$\underbrace{\hspace{10em}}_{2j \sin(3\hat{\omega}/2)}$

$$H(e^{j\hat{\omega}}) = 5j e^{-j3\hat{\omega}/2} \left(\sin(3\hat{\omega}/2) - \sin(\hat{\omega}/2) \right)$$

$$|H(e^{j\hat{\omega}})| = 5 \left| \sin(3\hat{\omega}/2) - \sin(\hat{\omega}/2) \right|$$

$$\text{at } \hat{\omega} = 0 \rightarrow H(e^{j\hat{\omega}}) = 0$$

$$\text{at } \hat{\omega} = \pi \rightarrow H(e^{j\hat{\omega}}) = 5|-1-1| = 10$$

$$\text{at } \hat{\omega} = \pi/2 \rightarrow H(e^{j\pi/2}) = 5|\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}| = 0$$

$\Rightarrow \boxed{C}$

$$(b) \quad y[n] = -0.8y[n-1] + x[n] - x[n-1]$$

$$H(e^{j\hat{\omega}}) = \frac{1 - e^{-j\hat{\omega}}}{1 + 0.8e^{-j\hat{\omega}}}$$

$$|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}}) H^*(e^{j\hat{\omega}}) = \frac{1 - e^{-j\hat{\omega}}}{1 + 0.8e^{-j\hat{\omega}}} \cdot \frac{1 - e^{j\hat{\omega}}}{1 + 0.8e^{j\hat{\omega}}}$$

$$= \frac{-e^{j\hat{\omega}} + 2 - e^{-j\hat{\omega}}}{0.8e^{j\hat{\omega}} + 1.64 + 0.8e^{-j\hat{\omega}}}$$

$$= \frac{2 - 2\cos\hat{\omega}}{1.64 + 1.6\cos\hat{\omega}}$$

$$\text{at } \hat{\omega} = \pi/2$$

$$H(\pi/2) = \sqrt{\frac{2-0}{1.64}}$$

$$= \sqrt{1.22} \approx 1.1$$

$$|H(e^{j\hat{\omega}})| = \sqrt{\frac{2 - 2\cos\hat{\omega}}{1.64 + 1.6\cos\hat{\omega}}}$$

$\boxed{B}$

$$\text{at } \hat{\omega} = 0, H(e^{j0}) = \sqrt{\frac{2-2}{1.64+1.6}} = 0$$

$$\text{at } \hat{\omega} = \pi, H(e^{j\pi}) = \sqrt{\frac{2-2(-1)}{1.64-1.6}} = \sqrt{\frac{4}{0.04}} = \sqrt{100} = 10$$