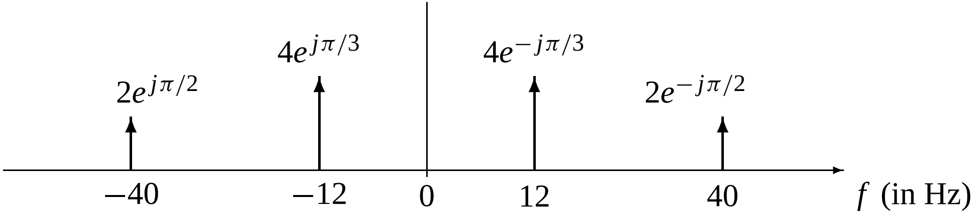
PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$. Make sure to express $x(t)$ as a real-valued signal.

(b) Is $x(t)$ a periodic signal? If so, what is its period?

(c) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $x(t) = 2e^{j\pi/2}e^{-j2\pi(40)t} + 4e^{j\pi/3}e^{-j2\pi(12)t}$

$+ 2e^{-j\pi/2}e^{j2\pi(40)t} + 4e^{-j\pi/3}e^{j2\pi(12)t}$

$= 4\cos(2\pi(40)t - \pi/2) + 8\cos(2\pi(12)t - \pi/3)$

(b) $x(t)$ is periodic. It is the sum of two terms that are periodic.

$4\cos(2\pi(40)t - \pi/2) \longrightarrow$ PERIOD $= \frac{1}{40}$ sec.

$8\cos(2\pi(12)t - \pi/3) \longrightarrow$ PERIOD $= \frac{1}{12}$ sec.

Find integers ℓ_1 & ℓ_2, so that

$\ell_1\left(\frac{1}{40}\right) = \ell_2\left(\frac{1}{12}\right)$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{40}{12} = \frac{10}{3} \Rightarrow \ell_1 = 10$ & $\ell_2 = 3$

Thus the common period is $\frac{\ell_1}{40} = \frac{10}{40} = \boxed{\frac{1}{4}\text{ sec}}$

(c) Sampling Theorem requires

$F_s > 2F_{HIGHEST} = 2(40\text{Hz}) = 80\text{Hz}$

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