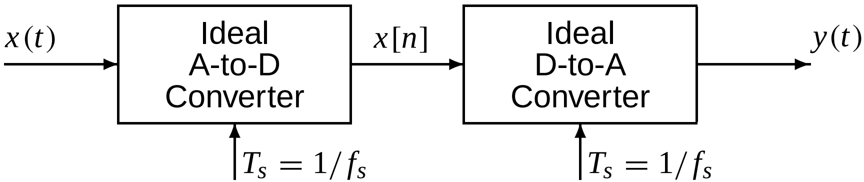
PROBLEM:

Consider the following system.



(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 14\cos(0.14\pi n - \pi/7)$$

If the sampling rate is $f_s = 8000$ samples/second, determine the output signal that will be heard. Give a formula for $y(t)$.

(b) If the input $x(t)$ is given by the chirp formula

$$x(t) = \cos(5000\pi t^2) \qquad \text{for } 0 \le t \le 2$$

determine the output signal that will be heard when $f_s = 8000$ samples/sec. Give a plot of instantaneous frequency versus time for $y(t)$.

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SOLUTION

(a) $x[n]$ has frequency $\hat{\omega}_0 = 0.14\pi$

Relationship between digital & analog freq is:

$$\hat{\omega} = 2\pi \frac{F_A}{F_s}$$

F_A = analog freq

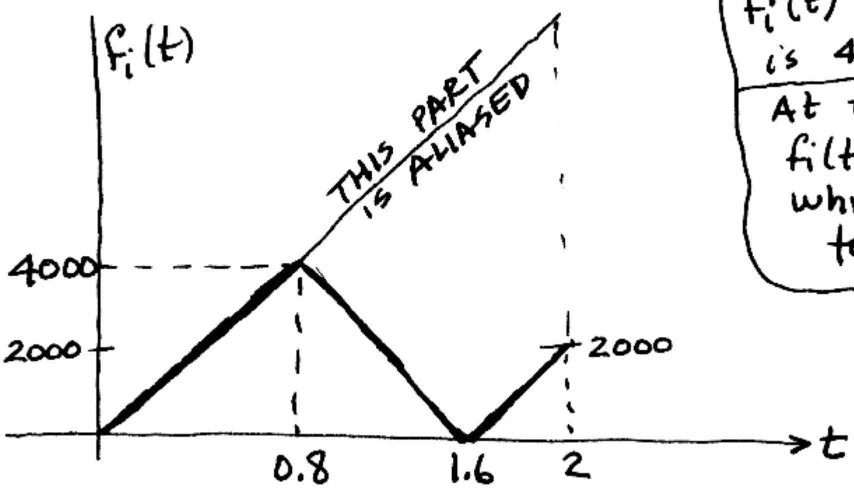
$$\Rightarrow F_A = \frac{\hat{\omega}}{2\pi} \cdot F_s = \frac{0.14\pi}{2\pi}(8000) = 0.07\,(8000)$$

$$\Rightarrow F_A = 560\,\text{Hz}$$

(b) $x(t) = \cos(5000\pi t^2) \qquad 0 \le t \le 2$

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(5000\pi t^2) = 5000t$$

As time t goes from $t=0$ to $t=2$, the instantaneous freq. goes from $f_i = 0$ to $f_i(t)\big|_{t=2} = 10{,}000\,\text{Hz}$. So there is aliasing.



$f_i(t)$ at $t=0.8$ is 4000 Hz

At $t = 1.6$ sec $f_i(t) = 8000$ Hz which aliases to $f=0$

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