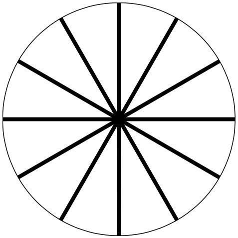
PROBLEM:

In old TV movies, all of us have seen the phenomenon where a spoked wagon wheel appears to move backwards. This is due to the 30 frames/sec sampling rate used in transmitting TV images. In the figure to the right, a twelve-spoked wheel is shown. Assume that the wheel is rotating *clockwise at a constant speed*. However, when seen on TV the spoke pattern of the wheel appears to to make a full 360° counter-clockwise revolution once every 60 frames (i.e., 2 seconds). Determine the rotation rate(s) that could have caused this illusion.



(a) Write a rotating phasor formula for the observed movement of an individual spoke. This should be a discrete-time signal formula that depends on n (the frame index).

(b) Derive a general equation that will give all possible rotation rates (in revs/sec).

(c) Evaluate your formula to find the $\boxed{\text{slowest}}$ rotation rate that produces the observation.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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SOLUTION

(a) Let $s[n]$ = spoke position

$$s[n] = e^{+j\,2\pi n/60}$$ ← starts at $0°$

(b) Let ω_0 = rotation rate of wheel in radians per sec.

Sample at 30 frames/sec.

$$e^{j\omega_0 t}\Big|_{t=n/30} = e^{j\,\omega_0 n/30}$$

$\boxed{\text{NOTE: we need } \omega_0 < 0 \text{ to get clockwise rotation}}$

Before we equate $s[n]$ to the sampled rotation, we must introduce the ambiguity of the twelve spokes. The digital freq. of $s[n]$ is $\hat{\omega}_0 = \frac{2\pi}{60}$

With twelve spokes the digital freq. could also be $\frac{2\pi}{60} + \frac{2\pi}{12}\ell$ where ℓ = integer

# of spokes

Now equate.

$$e^{j\,\omega_0 n/30} = e^{+j\left(\frac{2\pi}{60} + \frac{2\pi}{12}\ell\right)n}$$

$$\Rightarrow \frac{\omega_0}{30} = \frac{2\pi}{60} + \frac{2\pi\ell}{12}$$

$$\Rightarrow \omega_0 = 2\pi\left(\frac{30}{60} + \frac{30\ell}{12}\right) = 2\pi\left(\frac{1}{2} + \frac{5}{2}\ell\right).$$

ANSWER is $\left(\frac{1}{2} + \frac{5}{2}\ell\right)$ revs/sec. $\quad \ell = -1, -2, -3, \ldots$

(c) Need $\omega_0 < 0$, so take $\ell = -1$ to get slowest

$$\frac{1}{2} + \frac{5}{2}(-1) = -\frac{4}{2} \text{ rev/sec} \longrightarrow 2 \text{ rev/sec } \underline{\text{CLOCKWISE}}$$

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